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DOUBLE ARA–SHEHU TRANSFORM FOR PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. We study a mixed two-variable integral transform obtained by applying the ARA transform in one variable and the Shehu transform in the other. We identify the transform explicitly as a normalized and reparameterized double Laplace transform, establish boundedness, uniqueness, and an inversion formula on weighted function spaces, and derive concise rules for partial derivatives and double convolution. The formulation keeps ARA normalization and the Shehu scale parameter visible, which permits direct use of one-dimensional transform data for mixed initial-boundary conditions. A general algebraic representation is obtained for a class of second-order linear partial differential equations. Representative applications to a partial differential equation, a nonlinear integral equation, and a Volterra integro-differential equation illustrate the scope and limitations of the method.

Keywords. ARA transform, Shehu transform, double integral transform, partial differential equation, Volterra equation
2020 Mathematics Subject Classification. 44A05

1. Introduction

Integral transforms convert differential and integral operations into algebraic ones and are therefore useful for initial-boundary value problems. The ARA transform was introduced by Saadeh, Qazza, and Burqan [14], while the Shehu transform was introduced by Maitama and Zhao [13]. Related one-dimensional developments include the ARA treatment of fractional pantograph equations [9], the conformable Shehu decomposition method [12], and the classical Sumudu, natural, Kamal, and Aboodh transforms [18, 11, 17, 1].

Several two-variable constructions combine familiar kernels, including the double Mellin–ARA [3], Laplace–ARA [16], ARA–Sumudu [15], double Shehu [4], Laplace–Sawi [5], and Laplace–Shehu [10] transforms. Two further studies developed the double ARA–Sawi transform [2] and the double Sumudu–Shehu transform [6] for partial differential equations. They are particularly relevant here because they share, respectively, the ARA and Shehu components used in the present transform. Closely related work in this journal uses a modified double

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Laplace transform for partial derivatives [8]; integral transforms involving two-variable special functions are studied in [7].

The present transform is the tensor product of the ARA and Shehu kernels. We do not claim that this product creates an operator independent of the double Laplace transform. On the contrary, Proposition 3.1 shows the precise normalization and change of variables that connect them. The contribution is to (i) place the mixed transform on a weighted function space and state uniqueness and inversion conditions, (ii) retain the ARA normalization and Shehu scale explicitly when incorporating mixed boundary data, and (iii) derive one formula covering a class of second-order partial differential equations and representative integral and Volterra problems. This equivalence also makes the method's limitation transparent: it offers a convenient mixed notation and access to ARA and Shehu tables, but it does not surpass the double Laplace transform in general computational power.

2. ARA and Shehu transforms

In this section, we provide an overview and highlight key properties of the single transforms, namely the ARA and Shehu transforms.

2.1. The ARA transform.

Definition 2.1. Let $l(\nu)$ be a continuous function on $[0,\infty)$. Then The ARA transform of order m of $l(\nu)$ is expressed as follows:

$$A_m(l(\nu))(\alpha) = L(m,\alpha) = \alpha\int_0^\infty \nu^{m-1}e^{-\alpha\nu}l(\nu)d\nu,\ \ \alpha>0,\ \text{ for } m=1,2,3,....$$

In particular, if $m=1$, the ARA transform of order 1 is expressed as

$$A_1(l(\nu))(\alpha) = L(\alpha) = \alpha\int_0^\infty e^{-\alpha\nu}l(\nu)\,d\nu,\ \ \alpha>0.$$

In the rest of the study, we denote $A_1(l(\nu))(\alpha)$ by $A(l(\nu))(\alpha)$.

The following are some fundamental properties of the ARA transform.

Let $L(\alpha)=A(l(\nu))$, then for nonzero constants u and v, we have

$$A(ul_1(\nu)+vl_2(\nu)) = uA(l_1(\nu))+vA(l_2(\nu)), \tag{2.1}$$

where $l_1(\nu)$ and $l_2(\nu)$ are continuous functions on $(0,\infty)$.

$$A(\nu^u) = \frac{\Gamma(u+1)}{\alpha^u}, \tag{2.2}$$

$$A(e^{u\nu}) = \frac{\alpha}{\alpha-u},\ \ u\in\mathbb{R}, \tag{2.3}$$

$$A(l'(\nu)) = \alpha L(\alpha)-\alpha l(0), \tag{2.4}$$

$$A(l''(\nu)) = \alpha^2L(\alpha)-\alpha^2l(0)-\alpha l'(0). \tag{2.5}$$

2.2. The Shehu transform.

Definition 2.2. Let $g(\psi)$ be a continuous function on $[0,\infty)$. Then The Shehu transform of $g(\psi)$ is expressed as follows

$$G(\beta,\gamma) = H(g(\psi)) = \int_0^\infty e^{-\frac{\beta\psi}{\gamma}} g(\psi) d\psi.$$

The inverse Shehu transform is given by

$$g(\psi) = H^{-1}(G(\beta,\gamma)) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{1}{\gamma} e^{\frac{\beta\psi}{\gamma}} G(\beta,\gamma)\; d\beta.$$

Let us now explore the core properties that define the Shehu transform.

Suppose that $G_1(\beta,\gamma) = H(g_1(\psi))$ and $G_2(\beta,\gamma) = H(g_2(\psi))$,with u and v as nonzero real numbers, the following properties hold

$$H(ug_1(\psi) + vg_2(\psi)) = uH(g_1(\psi)) + vH(g_2(\psi)), \tag{2.6}$$

$$H(\psi^u) = \Gamma(u+1)\left(\frac{\gamma}{\beta}\right)^{u+1}, \tag{2.7}$$

$$H(e^{v\psi}) = \frac{\gamma}{\beta - v\gamma}, \tag{2.8}$$

$$H(g'(\psi)) = \frac{\beta}{\gamma} G(\beta,\gamma) - g(0), \tag{2.9}$$

$$H(g''(\psi)) = \frac{\beta^2}{\gamma^2} G(\beta,\gamma) - \frac{\beta}{\gamma} g(0) - g'(0). \tag{2.10}$$

3. The Double ARA-Shehu transform

For $a, b \in \mathbb{R}$, define the weighted space

$$X_{a,b} = L^1\left(\mathbb{R}_+^2, e^{-a\nu - b\psi}\, d\nu d\psi\right), \qquad \|f\|_{a,b} = \int_0^\infty\int_0^\infty e^{-a\nu-b\psi}|f(\nu,\psi)|\, d\nu d\psi.$$

For $f \in X_{a,b}$, $\operatorname{Re}\alpha \ge a$, and $\operatorname{Re}(\beta/\gamma) \ge b$, with $\alpha \ne 0$ and $\gamma \ne 0$, we define

$$\mathcal{D}f(\alpha,\beta,\gamma) = \alpha\int_0^\infty\int_0^\infty e^{-\alpha\nu-(\beta/\gamma)\psi} f(\nu,\psi)\, d\psi d\nu. \tag{3.1}$$

Proposition 3.1. *Let $\mathcal{L}_2 f(p,q)$ denote the double Laplace transform. Then*

$$\mathcal{D}f(\alpha,\beta,\gamma) = \alpha\,\mathcal{L}_2 f\left(\alpha, \frac{\beta}{\gamma}\right). \tag{3.2}$$

Consequently, $\mathcal{D}$ is linear and $|\mathcal{D}f(\alpha,\beta,\gamma)| \le |\alpha|\|f\|_{a,b}$ in the stated half-planes. It is holomorphic in α and in β/γ in their interiors.

Proof. Equation (3.2) follows by comparing the two kernels. The estimate follows from the definition of $\|f\|_{a,b}$, and holomorphy follows by dominated differentiation on compact subsets of the half-planes. $\square$

Theorem 3.2 (Uniqueness and inversion)**.** *Let $f \in X_{a,b}$ be locally of bounded variation and continuous at (ν, ψ). If $c > a$ and $d > b$, then, at every such continuity point,*

$$f(\nu, \psi) = \frac{1}{(2\pi i)^2} \int_{c-i\infty}^{c+i\infty} \int_{d-i\infty}^{d+i\infty} e^{p\nu+q\psi} \frac{\mathcal{D}f(p, q\gamma, \gamma)}{p} \, dq \, dp, \tag{3.3}$$

where the Bromwich integrals are understood as symmetric limits. Hence $\mathcal{D}$ is injective on this class. For general $f \in X_{a,b}$, the same formula holds in the standard L^1 inversion sense whenever the boundary transforms are integrable.

Proof. By Proposition 3.1,

$$\frac{\mathcal{D}f(p, q\gamma, \gamma)}{p} = \mathcal{L}_2 f(p, q).$$

Applying the one-dimensional Laplace inversion theorem successively in q and p gives (3.3); uniqueness follows from uniqueness of the double Laplace transform. □

Remark 3.3. The parameter γ is a scale parameter rather than a new spectral dimension: only the ratio β/γ occurs. Thus DAHT and the double Laplace transform have the same information content. DAHT is useful when ARA-normalized data in ν and Shehu-scaled data in ψ are already available, but it does not reduce the intrinsic complexity of every PDE.

If $f(\nu, \psi) = l(\nu)g(\psi)$, Fubini's theorem gives

$$\mathcal{D}f(\alpha, \beta, \gamma) = A(l)(\alpha)H(g)(\beta, \gamma). \tag{3.4}$$

This corrects the notation and states the precise separability condition.

3.1. **Derivative properties.** Here, we introduce some fundamental properties of the DAHT.

Let $F(\alpha, \beta, \gamma) = A_\nu H_\psi(f(\nu, \psi))$. Then

(1)
$$A_\nu H_\psi \left(\frac{\partial f(\nu, \psi)}{\partial \nu} \right) = \alpha F(\alpha, \beta, \gamma) - \alpha H(f(0, \psi)). \tag{3.5}$$

(2)
$$A_\nu H_\psi \left(\frac{\partial^2 f(\nu, \psi)}{\partial \nu^2} \right) = \alpha^2 F(\alpha, \beta, \gamma) - \alpha^2 H(f(0, \psi)) - \alpha H(f_\nu(0, \psi)).$$

(3)
$$A_\nu H_\psi \left(\frac{\partial f(\nu, \psi)}{\partial \psi} \right) = \frac{\beta}{\gamma} F(\alpha, \beta, \gamma) - A(f(\nu, 0)). \tag{3.6}$$

(4)
$$A_\nu H_\psi \left(\frac{\partial^2 f(\nu, \psi)}{\partial \psi^2} \right) = \frac{\beta^2}{\gamma^2} F(\alpha, \beta, \gamma) - \frac{\beta}{\gamma} A(f(\nu, 0)) - A(f_\psi(\nu, 0)). \tag{3.7}$$

(5)
$$A_\nu H_\psi\left(\frac{\partial^2 f(\nu,\psi)}{\partial\nu\partial\psi}\right)=\frac{\alpha\beta}{\gamma}F(\alpha,\beta,\gamma)-\alpha A(f(\nu,0))-\frac{\alpha\beta}{\gamma}H(f(0,\psi))+\alpha f(0,0). \tag{3.8}$$

Proof. Assume that the displayed boundary traces exist and that differentiation and integration may be interchanged. Integrating once in ν gives
$$\mathcal{D}(f_\nu)=\alpha\mathcal{D}f-\alpha H(f(0,\cdot)),$$
and integrating once in ψ gives
$$\mathcal{D}(f_\psi)=\frac{\beta}{\gamma}\mathcal{D}f-A(f(\cdot,0)).$$
Repeating the relevant identity yields the two pure second-derivative formulas. Applying the two first-derivative identities successively yields the mixed-derivative formula. Boundary terms at infinity vanish under the weighted integrability hypotheses of Theorem 3.2 applied to the relevant derivatives. □

3.2. **Double convolution.** For suitable f and k, define
$$(f**k)(\nu,\psi)=\int_0^\nu\int_0^\psi f(\nu-u,\psi-v)k(u,v)\,dv\,du. \tag{3.9}$$

Theorem 3.4. *If the integrals are absolutely convergent, then*
$$\mathcal{D}(f**k)(\alpha,\beta,\gamma)=\frac{1}{\alpha}\mathcal{D}f(\alpha,\beta,\gamma)\mathcal{D}k(\alpha,\beta,\gamma). \tag{3.10}$$

Proof. By Proposition 3.1 and the double Laplace convolution theorem,
$$\mathcal{D}(f**k)=\alpha\mathcal{L}_2(f**k)=\alpha(\mathcal{L}_2f)(\mathcal{L}_2k)=\alpha\frac{\mathcal{D}f}{\alpha}\frac{\mathcal{D}k}{\alpha}.$$
□

4. Applications

In this section, we use the DAHT for solving PDEs, IEs and Volterra Integro PDEs.

4.1. **DAHT for solving PDEs.** Consider the PDE of the form

$$D_1f_{\nu\nu}+D_2f_{\nu\psi}+D_3f_{\psi\psi}+D_4f_\nu+D_5f_\psi+D_6f\left(\nu,\psi\right)=k\left(\nu,\psi\right), \tag{4.1}$$

With ICs

$f(\nu,0)=l_1\left(\nu\right),\ f_\psi(\nu,0)=l_2\left(\nu\right),$

and BCs

$$f(0,\psi) = g_1(\psi),\ f_\nu(0,\psi) = g_2(\psi),$$

and assuming $f(0,0) = \Phi$.

Given that $f(\nu,\psi)$ is the unknown function, $k(\nu,\psi)$ is the source term, and $D_1, D_2, ..., D_6$ and Φ are constants, we aim to apply the DAHT to Equation (4.1).
To achieve this, we first apply the single ARA transform to the ICs and the single Shehu transform to the BCs.

$$\begin{aligned} A(l_1)(\alpha) &= L_1(\alpha), & A(l_2)(\alpha) &= L_2(\alpha), \\ H(g_1)(\beta,\gamma) &= G_1(\beta,\gamma), & H(g_2)(\beta,\gamma) &= G_2(\beta,\gamma). \end{aligned}$$

By applying the DAHT to Equation (4.1), we have

$$\begin{aligned} &D_1 A_\nu H_\psi(f_{\nu\nu}) + D_2 A_\nu H_\psi(f_{\nu\psi}) + D_3 A_\nu H_\psi(f_{\psi\psi}) + D_4 A_\nu H_\psi(f_\nu) \\ &\quad + D_5 A_\nu H_\psi(f_\psi) + D_6 A_\nu H_\psi(f(\nu,\psi)) = A_\nu H_\psi(k(\nu,\psi)). \end{aligned} \tag{4.2}$$

By the properties of the derivatives in Equations (3.5) − (3.8), we get

$$\begin{aligned} &D_1\left(\alpha^2 F(\alpha,\beta,\gamma) - \alpha^2 G_1(\beta,\gamma) - \alpha G_2(\beta,\gamma)\right) \\ &+D_2\left(\frac{\alpha\beta}{\gamma}F(\alpha,\beta,\gamma) - \alpha L_1(\alpha) - \frac{\alpha\beta}{\gamma}G_1(\beta,\gamma) + \alpha\Phi\right) \\ &+D_3\left(\frac{\beta^2}{\gamma^2}F(\alpha,\beta,\gamma) - \frac{\beta}{\gamma}L_1(\alpha) - L_2(\alpha)\right) + D_4\left(\alpha F(\alpha,\beta,\gamma) - \alpha G_1(\beta,\gamma)\right) \\ &+D_5\left(\frac{\beta}{\gamma}F(\alpha,\beta,\gamma) - L_1(\alpha)\right) + D_6 F(\alpha,\beta,\gamma) = K(\alpha,\beta,\gamma). \end{aligned} \tag{4.3}$$

Simplify Equation 4.3 as follows
Set

$$P = D_1\alpha^2 + D_2\frac{\alpha\beta}{\gamma} + D_3\frac{\beta^2}{\gamma^2} + D_4\alpha + D_5\frac{\beta}{\gamma} + D_6.$$

Then

$$\begin{aligned} F(\alpha,\beta,\gamma) = \frac{1}{P}\Bigg[&\left(D_1\alpha^2 + D_2\frac{\alpha\beta}{\gamma} + D_4\alpha\right)G_1 + D_1\alpha G_2 \\ &+\left(D_2\alpha + D_3\frac{\beta}{\gamma} + D_5\right)L_1 + D_3 L_2 - D_2\alpha\Phi + K\Bigg]. \end{aligned} \tag{4.4}$$

Example 4.1. Consider the heat equation

$$f_{\nu\nu} - f_{\psi} = -2f(\nu,\psi) + 4, \text{ where } \nu, \psi \geq 0,$$

With IC
$f(\nu, 0) = 2 - \cos\nu,$
and BCs
$f(0,\psi) = 2 - e^{\psi},\ f_{\nu}(0,\psi) = 0.$

Solution. By applying the single ARA transform to the IC and the single Shehu transform to the BCs, we obtain
$L_1 = 2 - \frac{\alpha^2}{\alpha^2+1},\ G_1 = \frac{2\gamma}{\beta} - \frac{\gamma}{\beta-\gamma},\ G_2 = 0$
and $K(\alpha,\beta,\gamma) = A_{\nu}H_{\psi}(4) = \frac{4\gamma}{\beta}.$
Substitute in Equation (4.4) $D_1 = 1,\ D_5 = -1,\ D_6 = 2,\ D_2 = D_3 = D_4 = 0$ and the values of L_1, G_1, G_2 and K, we get

$$\begin{aligned} F(\alpha,\beta,\gamma) &= \frac{\frac{2\alpha^2\gamma}{\beta} - \frac{\alpha^2\gamma}{\beta-\gamma} - 2 + \frac{\alpha^2}{\alpha^2+1} + \frac{4\gamma}{\beta}}{\alpha^2 - \frac{\beta}{\gamma} + 2} \qquad (4.5)\\ &= \frac{\frac{2\left(\alpha^2\gamma-\beta+2\gamma\right)}{\beta} - \frac{\alpha^2\left(\alpha^2\gamma-\beta+2\gamma\right)}{(\beta-\gamma)(\alpha^2+1)}}{\frac{\alpha^2\gamma-\beta+2\gamma}{\gamma}}\\ &= \frac{2\gamma}{\beta} - \frac{\alpha^2\gamma}{(\beta-\gamma)(\alpha^2+1)}. \end{aligned}$$

So,

$$f(\nu,\psi) = A_{\nu}^{-1}H_{\psi}^{-1}\left(\frac{2\gamma}{\beta} - \frac{\alpha^2\gamma}{(\beta-\gamma)(\alpha^2+1)}\right) = 2 - e^{\psi}\cos\nu.$$

Its graph is ☐

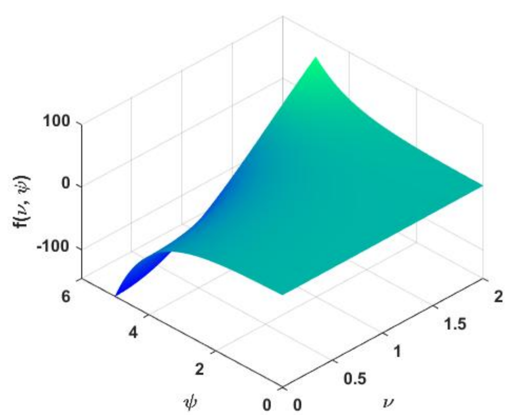

FIGURE 1. The solution of Example 4.1

4.2. DAHT for solving IEs.

Example 4.2. Consider the IE
$\int_0^{\nu}\int_0^{\psi} f(\nu-u,\psi-v)f(u,v)\,dv\,du = 4.$

Solution. By (3.9), we have

$$(f**f)(\nu,\psi) = 4. \tag{4.6}$$

Apply the DAHT to Equation (4.6), we get

$$A_\nu H_\psi\left((f**f)(\nu,\psi)\right) = A_\nu H_\psi(4).$$

By Theorem 3.4, we get

$$\frac{1}{\alpha}F(\alpha,\beta,\gamma)F(\alpha,\beta,\gamma) = \frac{4\gamma}{\beta}.$$

So,

$$F(\alpha,\beta,\gamma) = 2\sqrt{\frac{\alpha\gamma}{\beta}}.$$

Thus,

$$f(\nu,\psi) = A_\nu^{-1}H_\psi^{-1}\left(2\sqrt{\frac{\alpha\gamma}{\beta}}\right) = \frac{2}{\pi\sqrt{\nu\psi}}.$$

Its graph is □

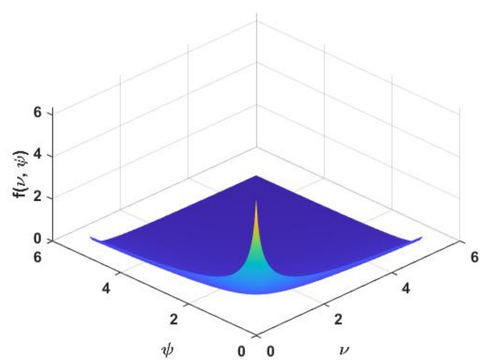

FIGURE 2. The solution of the nonlinear integral equation.

4.3. DAHT for solving Volterra Integro PDEs.

Example 4.3. Consider the equation of Volterra Integro PDE.

$$f_\nu + f_\psi = \sin\psi + \nu\cos\psi + \nu^2\cos\psi - \nu^2 + 2\int_0^{\nu}\int_0^{\psi} f(u,v)\,dv\,du, \text{ where } \nu,\psi \geq 0, \tag{4.7}$$

with $f(\nu,0) = 0$ and $f(0,\psi) = 0$.

Solution. By applying the single ARA transform and the single Shehu transform to the ICs, we get
$L_1 = 0$, $G_1 = 0$.
By (3.9) and Theorem 3.4, we have

$$\int_0^{\nu}\int_0^{\psi} f(u,v)\,dv\,du = (1 * *f)(\nu,\psi) = \frac{1}{\alpha} \times \frac{\gamma}{\beta} F(\alpha,\beta,\gamma). \tag{4.8}$$

Apply the DAHT to Equation (4.8), we get

$$\begin{aligned}\alpha F(\alpha,\beta,\gamma) + \frac{\beta}{\gamma} F(\alpha,\beta,\gamma) &= \frac{\gamma^2}{\beta^2+\gamma^2} + \frac{1}{\alpha} \times \frac{\beta\gamma}{\beta^2+\gamma^2} + \frac{2}{\alpha^2} \times \frac{\beta\gamma}{\beta^2+\gamma^2} \\ &\quad -\frac{2}{\alpha^2} \times \frac{\gamma}{\beta} + 2\frac{1}{\alpha} \times \frac{\gamma}{\beta} F(\alpha,\beta,\gamma).\end{aligned}$$

So,

$$\begin{aligned}\frac{\alpha^2\beta\gamma + \alpha\beta^2 - 2\gamma^2}{\alpha\beta\gamma} \times F(\alpha,\beta,\gamma) &= \\ &\quad \frac{\alpha^2\beta\gamma^2 + \alpha\beta^2\gamma + 2\beta^2\gamma - 2\gamma\left(\beta^2+\gamma^2\right)}{\alpha^2\beta\left(\beta^2+\gamma^2\right)}.\end{aligned}$$

Thus,

$$\begin{aligned}F(\alpha,\beta,\gamma) &= \frac{\gamma^2\left(\alpha^2\beta\gamma + \alpha\beta^2 - 2\gamma^2\right)}{\alpha\left(\beta^2+\gamma^2\right)\left(\alpha^2\beta\gamma + \alpha\beta^2 - 2\gamma^2\right)} \\ &= \frac{\gamma^2}{\alpha\left(\beta^2+\gamma^2\right)}.\end{aligned}$$

Therefore,

$$f(\nu,\psi) = A_{\nu}^{-1} H_{\psi}^{-1}\left(\frac{\gamma^2}{\alpha\left(\beta^2+\gamma^2\right)}\right) = \nu \sin\psi.$$

Its graph is □

5. Conclusion

The double ARA–Shehu transform is exactly a normalized double Laplace transform under the spectral substitution $q = \beta/\gamma$. Making this relation explicit clarifies both the method's theoretical basis and its limitations. On weighted L^1 spaces the transform is bounded and injective, and the usual double Bromwich formula supplies its inverse under standard regularity conditions. Its practical value is representational: it preserves ARA normalization in one variable and Shehu scaling in the other, so mixed initial and boundary data can be inserted from the corresponding one-dimensional transform tables. The three retained examples represent a PDE, a nonlinear integral equation, and a Volterra integro-differential equation. Further work would need genuinely new operator estimates

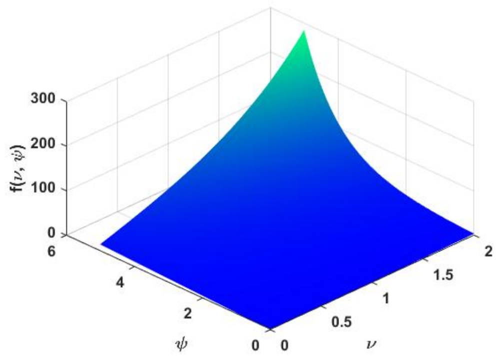

Figure 3. The solution of the Volterra integro-differential equation.

or applications in which the Shehu scaling parameter has a demonstrable analytic or numerical benefit.

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Conflict of interest

The authors declare no conflict of interest.

Data availability

No data sets were generated or analyzed in this study.

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