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A NOVEL APPROACH TO SOLVING THE TIME-FRACTIONAL FITZHUGH-NAGUMO MODEL

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Abstract. The main goal of the study is to look at the fractional model of the Fitzhugh-Nagumo model using a reliable and precise numerical method that combines Adomian decomposition with the Shehu transform. Adomian decomposition is used to reduce nonlinear terms. Both the convergence and analysis of errors in the suggested method are described. Graphical illustrations of numerical results show the efficiency of the suggested technique. The Caputo derivative is used since it facilitates the incorporation of conventional starting and boundary conditions in the problem formulation. Our study has numerous implications in various areas of engineering and science and could be considered an alternative to existing methodologies. The results also show that the approach is valid and has the potential to be an effective way to arrive at approximate solutions to the Fitzhugh-Nagumo model.

Keywords. Fractional Fitzhugh-Nagumo model, Adomian decomposition method, Shehu transform , Caputo fractional derivative.
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1. Introduction

Recently, the Fitzhugh-Nagumo model has attracted considerable interest from scientists and mathematicians because of its essential significance in mathematical physics. This equation is applicable in various domains, including flame propagation, exponential population growth, neuroscience, branching Brownian movement mechanisms, automatic reactions, and reactor theory [1, 2, 3].
Several researchers have focused on the utility and importance of fractional-order integrals and derivatives in mathematical modeling throughout various scientific and engineering fields [4, 20]. Researchers have consistently studied this field due to its importance in scientific applications, and they describe the Fitzhugh-Nagumo model for time fractional order with the formula [9].

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$$\mathcal{D}_\tau^\delta \varphi(\mathcal{V},\tau) = \lambda(\tau)\frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2} - \varpi(\tau)\frac{\partial\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}} + \zeta(\tau)\varphi(\mathcal{V},\tau)(\varphi(\mathcal{V},\tau)-\gamma)(1-\varphi(\mathcal{V},\tau)), \tag{1.1}$$

where $\mathcal{D}_\tau^\delta = \frac{\partial^\delta}{\partial\tau^\delta}$ is fractional derivative in Caputo sense, $\varphi(\mathcal{V},\tau)$ is the function of space $\mathcal{V}$ and time τ , γ is an arbitrary constant and $\lambda(\tau),\ \varpi(\tau),\ \zeta(\tau)$ are real value function in τ. In this article we study the classic Fitzhugh-Nagumo model for $\lambda(\tau) = \zeta(\tau) = 1$ and $\varpi(\tau) = 0$.

$$\mathcal{D}_\tau^\delta \varphi(\mathcal{V},\tau) = \frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2} + \varphi(\mathcal{V},\tau)(\varphi(\mathcal{V},\tau)-\gamma)(1-\varphi(\mathcal{V},\tau)), \delta \in (0,1], \tau > 0, \mathcal{V} \in \Omega \tag{1.2}$$

With the initial condition

$$\varphi(\mathcal{V},0) = \varphi_0(\mathcal{V}), \tag{1.3}$$

If $\gamma = -1$ The Equation (1.2) goes to nonlinear fractional well-known Newell-Whitehead

$$\mathcal{D}_\tau^\delta \varphi(\mathcal{V},\tau) = \frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2} + \varphi(\mathcal{V},\tau)(\varphi(\mathcal{V},\tau)+1)(1-\varphi(\mathcal{V},\tau)), \tau > 0, \mathcal{V} \in \Omega. \tag{1.4}$$

The Equation (1.2) given fractional nonlinear Fitzhugh-Nagumo equation (FNE) if $\gamma = 1$ [7, 18],

$$\mathcal{D}_\tau^\delta \varphi(\mathcal{V},\tau) = \frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2} + \varphi(\mathcal{V},\tau)(\varphi(\mathcal{V},\tau)-1)(1-\varphi(\mathcal{V},\tau)), \tau > 0, \mathcal{V} \in \Omega. \tag{1.5}$$

And if $\gamma = 0$, that given Fisher's equation of fractional derivative order

$$\mathcal{D}_\tau^\delta \varphi(\mathcal{V},\tau) = \frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2} + \varphi^2(\mathcal{V},\tau)(1-\varphi(\mathcal{V},\tau)), \tau > 0, \mathcal{V} \in \Omega. \tag{1.6}$$

Recently, various numerical and analytical methods have been employed to tackle classical and fractional (FNE) equations to obtain their analytic and semi-analytic solutions. These methods include the homotopy analysis method [6], the iterative method combined with power series and the Mohand transform [7], the conformable fractional method paired with the Adomian decomposition method and the Sumudu transform [8], the q-homotopy method in conjunction with the Laplace transform [9], and the residual power and new iteration methods alongside the Laplace transform [10], among others.

This research proposes a combined approach utilizing the Shehu transform and the Adomian decomposition method to examine the fractional model of the Fitzhugh-Nagumo model. The Adomian decomposition method [11, 12] is a significant technique used to solve differential equations introduced by George Adomian; however, it is difficult to handle the fractional order components. Here comes the role of the Shehu transform to decompose the fractional order, which can be considered a powerful tool for addressing distinct nonlinear problems and has already shown efficacy in resolving numerous linear and nonlinear differential

equations. The Adomian decomposition method, when integrated with the Shehu transform, yields a more efficient and straightforward approach for addressing numerous nonlinear problems in science, relative to alternative mathematical techniques. One of the primary benefits of employing the suggested method is that typically only some terms are enough for getting a sufficiently precise solution.
The following parts of the article are structured as follows: In section 2 We introduced some preliminary concepts and the definition of fractional calculus and the Shehu transform. In Section 3, we will discuss the treatment using the Shehu transform Adomain decomposition method (STADM). In section 4 we study some theorems that give existence and uniqueness of the solution for the present method. In Section 5, we provide an example of the time-fractional Fitzhugh-Nagumo model. In section 6, we discuss the results. Finally, in the section 7, our conclusion is given.

2. Fundamental Preliminary

This part provides essential concepts and basic knowledge to fractional calculus and the Shehu transform, that is utilized later in this article, for more information see [14]

Definition 2.1. The left Caputo derivative of the continuous function $\mathcal{F}:[0,\mathcal{T}]\to\mathbb{R}$ of order $\delta\geq 0$ has been defined with

$$\mathcal{D}^{\delta}\mathcal{F}(\tau)=\begin{cases}\frac{1}{\Gamma(n-\delta)}\int_0^{\tau}(\tau-\xi)^{n-\delta-1}\mathcal{F}^{(n)}(\xi)d\xi & \text{for } n-1<\delta\leq n\\ \frac{\partial^n}{\partial\tau^n}\mathcal{F}(\tau) & \text{for } n=\delta\end{cases}$$

where the $\Gamma(.)$ is the well-known Euler Gamma function.

Definition 2.2. The relation between the fractional integral Riemann-Liouville and Caputo derivative to the function
$\mathcal{F}(\tau)$ has been defined with

$$\mathcal{I}^{\delta}\mathcal{D}^{\delta}\mathcal{F}(\tau)=\mathcal{F}(\tau)-\sum_{m=0}^{n-1}\mathcal{F}^{(m)}(0^+)\frac{\tau^m}{m!},\tau>0.$$

Definition 2.3. [13] The function $\mathcal{Q}(\tau)$ of exponential order is defined over the set function,

$$\mathcal{U}=\left\{\mathcal{Q}(\tau),\ \exists\mathbb{N},\upsilon_0,\upsilon_1>0,|\mathcal{Q}(\tau)|<\mathbb{N}exp\left(\frac{|\tau|}{\upsilon_i}\right),\text{if } \tau\in(-1)^j\times[0,\infty)\right\},$$

the shehu integral transform defined by

$$\mathcal{S}[\mathcal{Q}(\tau)]=\mathscr{F}(s,v)=\int_0^{\infty}exp\left(-\frac{st}{v}\right)\mathcal{Q}(\tau)d\tau,\tau>0. \tag{2.1}$$

Shehu transform inverse denoted by $\mathcal{S}^{-1}$,

$$\mathcal{S}^{-1}[\mathscr{F}(s,v)]=\mathcal{Q}(\tau),\text{for } \tau>0.$$

equivalent to,

$$\mathcal{Q}(\tau)=\mathcal{S}^{-1}[\mathscr{F}(s,v)]=\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty}\frac{1}{v}exp\left(\frac{st}{v}\right)[\mathscr{F}(s,v)]ds.$$

Theorem 2.4. [13] *The shehu transform for Caputo fractional derivative $\mathcal{D}_{\tau}^{\delta}\varphi(\mathcal{V},\tau)$ given by*

$$\mathcal{S}[\mathcal{D}_{\tau}^{\delta}\varphi(\mathcal{V},\tau)]=\frac{s^{\delta}}{v^{\delta}}\mathcal{S}[\varphi(\mathcal{V},\tau)]-\sum_{i=0}^{n-1}\left(\frac{s}{v}\right)^{\delta-i-1}\frac{\partial^{i}}{\partial\tau^{i}}\varphi(\mathcal{V},0),n-1<\delta\leq n,n\in\mathbb{N}. \tag{2.2}$$

Proposition 2.5. [13] *Some properties for Shehu transform*

(1) *Property of Linearity, Let two function $\mu\mathcal{F}(\tau)$ and $\lambda\mathcal{H}(\tau)\in\mathcal{A}$, then $(\mu\mathcal{F}(\tau)+\lambda\mathcal{H}(\tau))\in\mathcal{A}$*
$\mathcal{S}[\mu\mathcal{F}(\tau)+\lambda\mathcal{H}(\tau)]=\mu\mathcal{S}[\mathcal{F}(\tau)]+\lambda\mathcal{S}[\mathcal{H}(\tau)]$, *where μ, λ are nonzero constant.*
(2) $\mathcal{S}[1]=\frac{v}{s}$
(3) $\mathcal{S}[\tau]=\frac{v^2}{s^2}$
(4) $\mathcal{S}\left[\frac{\tau^{\delta}}{\Gamma(\delta+1)}\right]=\left(\frac{v}{s}\right)^{\delta+1},\delta=0,1,2,\ldots$

3. Shehu Adomain Decomposition Method

This portion of the research demonstrates the implementation of the Adomian decomposition technique in conjunction with the Shehu transform.
Let's assume that the PDE has a fractional order $\delta\in(0,1]$ of time,

$$\mathcal{D}_{\tau}^{\delta}\varphi(\mathcal{V},\tau)+\mathcal{K}(\varphi(\mathcal{V},\tau))+\mathscr{N}(\varphi(\mathcal{V},\tau))=\mathscr{G}(\mathcal{V},\tau), \tag{3.1}$$

subject to the initial condition

$$\varphi(\mathcal{V},0)=\varphi_0(\mathcal{V}), \tag{3.2}$$

where $\mathcal{D}_{\tau}^{\delta}=\frac{\partial^{\delta}}{\partial\tau^{\delta}}$ the fractional derivative in Caputo sense, $\mathcal{K}$ and $\mathscr{N}$ are linear and nonlinear operators, respectively. $\mathscr{G}$ is source function in $\mathcal{V}$ and τ, now let we apply the Shehu transform for Equation (3.1) we get,

$$\mathcal{S}[\mathcal{D}_{\tau}^{\delta}\varphi(\mathcal{V},\tau)+\mathcal{K}(\varphi(\mathcal{V},\tau))+\mathscr{N}(\varphi(\mathcal{V},\tau))]=\mathcal{S}[\mathscr{G}(\mathcal{V},\tau)], \tag{3.3}$$

By using the Equation (2.2)

$$\frac{s^{\delta}}{v^{\delta}}\mathcal{S}[\varphi(\mathcal{V},\tau)]-\sum_{c=0}^{n-1}\left(\frac{s}{v}\right)^{\delta-c-1}\frac{\partial^{c}}{\partial\tau^{c}}\varphi(\mathcal{V},0)=\mathcal{S}[\mathscr{G}(\mathcal{V},\tau)-\mathcal{K}(\varphi(\mathcal{V},\tau))-\mathscr{N}(\varphi(\mathcal{V},\tau))] \tag{3.4}$$

In this stage, Shehu transform converts a fractional derivative to a formula for algebra, utilizing the provided initial condition, we are capable of rewriting Equation (3.4)

$$\frac{s^{\delta}}{v^{\delta}}\mathcal{S}[\varphi(\mathcal{V},\tau)]=\left(\frac{s}{v}\right)^{\delta-1}\varphi(\mathcal{V},0)+\mathcal{S}[\mathscr{G}(\mathcal{V},\tau)-\mathcal{K}(\varphi(\mathcal{V},\tau))-\mathscr{N}(\varphi(\mathcal{V},\tau))], \tag{3.5}$$

$$\mathcal{S}[\varphi(\mathcal{V},\tau)] = \left(\frac{v}{s}\right)\varphi(\mathcal{V},0) + \left(\frac{v}{s}\right)^{\delta}\mathcal{S}[\mathscr{G}(\mathcal{V},\tau) - \mathcal{K}(\varphi(\mathcal{V},\tau)) - \mathscr{N}(\varphi(\mathcal{V},\tau))], \quad (3.6)$$

Based with the linearity property exhibited by Shehu transform, the Equation (3.6) transforms into

$$\mathcal{S}[\varphi(\mathcal{V},\tau)] = \left(\frac{v}{s}\right)\varphi(\mathcal{V},0)+\left(\frac{v}{s}\right)^{\delta}\mathcal{S}\left[\mathscr{G}(\mathcal{V},\tau)\right]-\left(\frac{v}{s}\right)^{\delta}\mathcal{S}\left[\mathcal{K}(\varphi(\mathcal{V},\tau)) + \mathscr{N}(\varphi(\mathcal{V},\tau))\right], \quad (3.7)$$

Utilize the $\mathcal{S}^{-1}$ for the Equation (3.7) to derive

$$\varphi(\mathcal{V},\tau) = \varphi(\mathcal{V},0)+\mathcal{S}^{-1}\left[\left(\frac{v}{s}\right)^{\delta}\mathcal{S}\left[\mathscr{G}(\mathcal{V},\tau)\right]\right]-\mathcal{S}^{-1}\left[\left(\frac{v}{s}\right)^{\delta}\mathcal{S}\left[\mathcal{K}(\varphi(\mathcal{V},\tau)) + \mathscr{N}(\varphi(\mathcal{V},\tau))\right]\right], \quad (3.8)$$

To decompose that nonlinear term in Equation (3.8) construct a series solution using the suggested technique, ADM is applied. The ADM shows a solution as an infinite series of linear terms as

$$\varphi(\mathcal{V},\tau) = \sum_{m=0}^{\infty}\varphi_m(\mathcal{V},\tau), \quad (3.9)$$

while the nonlinear terms in the ADM can be expanded in terms of the Adomian polynomial in the following way

$$\mathscr{N}(\varphi(\mathcal{V},\tau)) = \sum_{m=0}^{\infty}\mathbb{A}_m(\varphi_0,\varphi_1,\ldots,\varphi_m), \quad (3.10)$$

where,

$$\mathbb{A}_m(\varphi_0,\varphi_1,\ldots,\varphi_m) = \frac{1}{m!}\frac{d^m}{d\beta^m}\left[\mathscr{N}\left(\sum_{j=0}^{m}\beta^j\varphi_j\right)\right]_{\beta=0}, m = 0,1,2,\ldots \quad (3.11)$$

Using the Equation (3.9) and Equation (3.10) that we get in Equation (3.8)

$$\sum_{m=0}^{\infty}\varphi_m(\mathcal{V},\tau) = \varphi(\mathcal{V},0)+\mathcal{S}^{-1}\left[\left(\frac{v}{s}\right)^{\delta}\mathcal{S}\left[\mathscr{G}(\mathcal{V},\tau)\right]\right]-\mathcal{S}^{-1}\left[\left(\frac{v}{s}\right)^{\delta}\mathcal{S}\left[\mathcal{K}\left(\sum_{m=0}^{\infty}\varphi_m(\mathcal{V},\tau)\right) + \sum_{m=0}^{\infty}\mathbb{A}_m(\varphi)\right]\right] \quad (3.12)$$

To get the part solutions of the presented technique, we equate the terms on each side of Equation (3.12), resulting in the next relation

$$\varphi_0(\mathcal{V},\tau) = \varphi(\mathcal{V},0) + \mathcal{S}^{-1}\left[\left(\frac{v}{s}\right)^{\delta}\mathcal{S}\left[\mathscr{G}(\mathcal{V},\tau)\right]\right]$$

$$\varphi_{m+1}(\mathcal{V},\tau) = -\mathcal{S}^{-1}\left[\frac{v^{\delta}}{s^{\delta}}\mathcal{S}\left[\mathcal{K}(\varphi_m(\mathcal{V},\tau)) + \mathbb{A}_m(\varphi)\right]\right], m \geq 0 \quad (3.13)$$

After obtaining to the approximate solution of the component, we get the general solution in the following format as

$$\varphi(\mathcal{V},\tau)=\sum_{m=0}^{\infty}\varphi_m(\mathcal{V},\tau)=\varphi_0(\mathcal{V},\tau)+\varphi_1(\mathcal{V},\tau)+\varphi_2(\mathcal{V},\tau)+\dots \tag{3.14}$$

Finding the infinite component of the solution is difficult; therefore, after $(M)^{th}$ truncations, the iteration formula for (STADM) is as detailed below

$$\varphi(\mathcal{V},\tau)=\sum_{m=0}^{M}\varphi_m(\mathcal{V},\tau), \tag{3.15}$$

as $M\to\infty$ we can obtain an accurately approximate solution of Equation (3.3).

4. Convergence analysis

We study an appropriate scenario that ensures the FN model unique solution, we give the error analysis of the proposed technique.

Theorem 4.1. *Suppose that* $\mathbb{E}=(\mathcal{C}[\mathcal{I}],\|.\|)$ *be Banach space with the norm* $\|\varphi(\tau)\|=\max_{\tau\in\mathcal{I}}|\varphi(\tau)|$ *for all continuous function on* $\mathcal{I}=[0,T]$. $\mathrm{F}:\mathbb{E}\to\mathbb{E}$ *is a nonlinear mapping, then the result of* (3.12) *is unique for STADM as* $0<\Phi<1$, *where* $\Phi=(\mu_0+\mu_1)\left(\frac{\tau^\delta}{\Gamma(\delta+1)}\right)$

Proof. The solution of the nonlinear PDE of fractional order (3.3) by STADM given by,

$$\varphi_{m+1}(\mathcal{V},\tau)=\varphi_m(\mathcal{V},\tau)-\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left[\mathcal{K}(\varphi_m(\mathcal{V},\tau))+\mathcal{N}(\varphi_m(\mathcal{V},\tau))-\mathcal{G}(\mathcal{V},\tau)\right]\right],m\geq 0 \tag{4.1}$$

Suppose that $\mathcal{K},\mathcal{N}$ are Lipschitzian with $|\mathcal{K}(\varphi)-\mathcal{K}(\varphi^*)|<\mu_0|\varphi-\varphi^*|$ and $|\mathcal{N}(\varphi)-\mathcal{N}(\varphi^*)|<\mu_1|\varphi-\varphi^*|$, where μ_0,μ_1 are Lipschitz constant and $\varphi:=\varphi(\mathcal{V},\tau),\varphi^*:=\varphi^*(\mathcal{V},\tau)$ two different function

$$
\begin{aligned}
\|\mathrm{F}\varphi - \mathrm{F}\varphi^*\| &\leq \max_{\tau\in\mathcal{I}} \left| \mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left[\mathcal{K}(\varphi_m) - \mathcal{K}(\varphi_m^*) + \mathscr{N}(\varphi_m) - \mathscr{N}(\varphi_m^*)\right]\right]\right| \\
&\leq \max_{\tau\in\mathcal{I}} \left[\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left|\mathcal{K}(\varphi_m) - \mathcal{K}(\varphi_m^*)\right|\right] + \mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left|\mathscr{N}(\varphi_m) - \mathscr{N}(\varphi_m^*)\right|\right]\right] \\
&\leq \max_{\tau\in\mathcal{I}} \left[\mu_0\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left|\varphi - \varphi^*\right|\right] + \mu_1\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left|\varphi - \varphi^*\right|\right]\right] \\
&\leq \max_{\tau\in\mathcal{I}}(\mu_0+\mu_1)\left[\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left|\varphi - \varphi^*\right|\right]\right] \\
&= \max_{\tau\in\mathcal{I}}(\mu_0+\mu_1)\left[\left|\varphi - \varphi^*\right|\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}[1]\right]\right] \\
&= \max_{\tau\in\mathcal{I}}(\mu_0+\mu_1)\left[\left|\varphi - \varphi^*\right|\frac{\tau^\delta}{\Gamma(\delta+1)}\right] \\
&\leq (\mu_0+\mu_1)\left\|\varphi - \varphi^*\right\|\left[\frac{\tau^\delta}{\Gamma(\delta+1)}\right] \\
&= \Phi\left\|\varphi - \varphi^*\right\|
\end{aligned}
$$

For $0 < \Phi < 1$, by the Definition of contraction [15, 16] and By the well-known Banach Fixed Point Theorem [15, 16], hence the solution of Equation (4.1) is unique.

□

Theorem 4.2. *Let we consider $\{\varphi_m\}_{i=0}^m$ sequence in $\mathbb{E} = (\mathcal{C}[\mathcal{I}], \|.\|)$, if φ_m is Cauchy sequence $\forall n \in \mathbb{N}$, Then the result of (4.1) is convergent*

Proof. Suppose that $\varphi_m = \sum_{i=0}^m \varphi_i(\mathcal{V},\tau)$, to prove that φ_m Cauchy sequence in the Banach space $\mathbb{E} = (\mathcal{C}[\mathcal{I}], \|.\|)$

$$
\begin{aligned}
\|\varphi_m - \varphi_n\| &= \max_{\tau\in\mathcal{I}}\left|\sum_{i=n+1}^{m}\varphi_i\right| \\
&\leq \max_{\tau\in\mathcal{I}}\left|\mathcal{S}^{-1}\left[\frac{v\delta}{s^\delta}\mathcal{S}\left[\sum_{i=n+1}^{m}\left(\mathcal{K}(\varphi_{i-1}) + \mathscr{N}(\varphi_{i-1})\right)\right]\right]\right| \\
&= \max_{\tau\in\mathcal{I}}\left|\mathcal{S}^{-1}\left[\frac{v\delta}{s^\delta}\mathcal{S}\left[\sum_{i=n+1}^{m-1}\left(\mathcal{K}(\varphi_{i}) + \mathscr{N}(\varphi_{i})\right)\right]\right]\right| \\
&\leq \max_{\tau\in\mathcal{I}}\left|\mathcal{S}^{-1}\left[\frac{v\delta}{s^\delta}\mathcal{S}\left[\left(\mathcal{K}(\varphi_{m-1}) - \mathcal{K}(\varphi_{n-1}) + \mathscr{N}(\varphi_{m-1}) - \mathscr{N}(\varphi_{n-1})\right)\right]\right]\right| \\
&\leq \max_{\tau\in\mathcal{I}}(\mu_0+\mu_1)\left(\mathcal{S}^{-1}\left[\frac{v\delta}{s^\delta}\mathcal{S}\left[\left|\mathcal{K}(\varphi_{m-1}) - \mathcal{K}(\varphi_{n-1})\right|\right]\right] + \mathcal{S}^{-1}\left[\frac{v\delta}{s^\delta}\mathcal{S}\left[\left|\mathscr{N}(\varphi_{m-1}) - \mathscr{N}(\varphi_{n-1})\right|\right.\right.\right. \\
&= (\mu_0+\mu_1)\left[\frac{\tau^\delta}{\Gamma(\delta+1)}\right]\left\|\varphi_{m-1} - \varphi_{n-1}\right\| \\
&= \Phi\left\|\varphi_{m-1} - \varphi_{n-1}\right\|
\end{aligned}
$$

Let $m = n + 1$, then

$$\|\varphi_{n+1} - \varphi_n\| \leq \Phi \|\varphi_n - \varphi_{n-1}\| \leq \ldots \leq \Phi^n \|\varphi_1 - \varphi_0\|, \tag{4.2}$$

we obtain,

$$\begin{aligned} \|\varphi_m - \varphi_n\| &\leq \|\varphi_{n+1} - \varphi_n\| + \|\varphi_{n+2} - \varphi_{n+1}\| + \ldots + \|\varphi_m - \varphi_{m-1}\| \\ &\leq \left(\Phi^n + \Phi^{n+1} + \ldots + \Phi^{m-1}\right) \|\varphi_1 - \varphi_0\| \\ &\leq \Phi^n \left(\frac{1 - \Phi^{m-n}}{1 - \Phi}\right) \|\varphi_1\| \end{aligned} \tag{4.3}$$

For $\Phi \in (0, 1)$ we obtain that $1 - \Phi^{m-n} < 1$, hence we get

$$\|\varphi_m - \varphi_n\| \leq \frac{\Phi^n}{1 - \Phi} \max_{\tau \in \mathcal{I}} \|\varphi_1\|, \tag{4.4}$$

Because $\|\varphi_1\| < \infty$ and $\|\varphi_m - \varphi_n\| \to 0$, as $n \to \infty$, hence φ_m is Cauchy sequence in $\mathbb{E}$. □

Theorem 4.3. *The maximum absolute error of the predicted approximation generated via STADM is*

$$\left\| \varphi(\mathcal{V}, \tau) - \sum_{k=0}^{n} \varphi_k(\mathcal{V}, \tau) \right\| \leq \frac{\Phi^n}{1 - \Phi} \max_{\tau \in \mathcal{I}} \|\varphi_1\|.$$

Proof. From Theorem 4.2 we reached to the Equation (4.4)

$$\|\varphi_m - \varphi_n\| \leq \frac{\Phi^n}{1 - \Phi} \max_{\tau \in \mathcal{I}} \|\varphi_1\|, \tag{4.5}$$

as $m \to \infty$, hence $\varphi_m \to \varphi(\mathcal{V}, \tau)$ the we obtain

$$\|\varphi(\mathcal{V}, \tau) - \varphi_n\| \leq \frac{\Phi^n}{1 - \Phi} \max_{\tau \in \mathcal{I}} \|\varphi_1\| \tag{4.6}$$

That's obviously the maximum absolute error of the predicted approximation generated via STADM is

$$\left\| \varphi(\mathcal{V}, \tau) - \sum_{k=0}^{n} \varphi_k(\mathcal{V}, \tau) \right\| \leq \frac{\Phi^n}{1 - \Phi} \max_{\tau \in \mathcal{I}} \|\varphi_1\|.$$

□

5. Utilizing the STADM

Example 5.1. Consider the Fitzhugh-Nagumo model is give by

$$\mathcal{D}_\tau^\delta \varphi(\mathcal{V}, \tau) = \frac{\partial^2 \varphi(\mathcal{V}, \tau)}{\partial \mathcal{V}^2} - \varphi^3(\mathcal{V}, \tau) + (1 + \gamma)\varphi^2(\mathcal{V}, \tau) - \gamma\varphi(\mathcal{V}, \tau), \delta \in (0, 1] \tag{5.1}$$

Subject to the initial condition

$$\varphi(\mathcal{V},0)=\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}\mathcal{V}}{4}\right) \tag{5.2}$$

Utilizing the Shehu transform for both side for the Equation (5.1)

$$\mathcal{S}\left[\mathcal{D}_\tau^\delta\varphi(\mathcal{V},\tau)\right]=\mathcal{S}\left[\frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2}-\varphi^3(\mathcal{V},\tau)+(1+\gamma)\varphi^2(\mathcal{V},\tau)-\gamma\varphi(\mathcal{V},\tau)\right] \tag{5.3}$$

By using the property of Shehu transform and the initial value (5.2)

$$\mathcal{S}\left[\varphi(\mathcal{V},\tau)\right]=\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}\mathcal{V}}{4}\right)+\mathcal{S}\left[\frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2}-\varphi^3(\mathcal{V},\tau)+(1+\gamma)\varphi^2(\mathcal{V},\tau)-\gamma\varphi(\mathcal{V},\tau)\right] \tag{5.4}$$

Following that, we utilize the $\mathcal{S}^{-1}$

$$\varphi(\mathcal{V},\tau)=\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}\mathcal{V}}{4}\right)+\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left[\frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2}-\varphi^3(\mathcal{V},\tau)+(1+\gamma)\varphi^2(\mathcal{V},\tau)-\gamma\varphi(\mathcal{V},\tau)\right]\right] \tag{5.5}$$

Let us assume that,

$$\varphi(\mathcal{V},\tau)=\sum_{m=0}^{\infty}\varphi_m,\ \varphi^3(\mathcal{V},\tau)=\sum_{m=0}^{\infty}\mathbb{A}_m(\varphi),\ \varphi^2(\mathcal{V},\tau)=\sum_{m=0}^{\infty}\mathbb{B}_m(\varphi)$$

Where,

$$\begin{aligned}
&\mathbb{A}_0(\varphi)=\varphi_0^3 \text{ and } \mathbb{B}_0(\varphi)=\varphi_0^2\\
&\mathbb{A}_1(\varphi)=3\varphi_0^2\varphi_1 \text{ and } \mathbb{B}_1(\varphi)=2\varphi_0\varphi_1\\
&\mathbb{A}_2(\varphi)=3\varphi_0\varphi_1^2+3\varphi_0^2\varphi_2 \text{ and } \mathbb{B}_3(\varphi)=\varphi_1^2+2\varphi_0\varphi_2
\end{aligned}$$

And so on, in such a manner that

$$\begin{aligned}
\varphi_0(\mathcal{V},\tau)&=\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\sqrt{2}\mathcal{V}}{4}\right)\\
\varphi_1(\mathcal{V},\tau)&=\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left[\frac{\partial^2\varphi_0(\mathcal{V},\tau)}{\partial\mathcal{V}^2}-\mathbb{A}_0(\varphi)+(1+\gamma)\mathbb{B}_0(\varphi)-\gamma\varphi_0(\mathcal{V},\tau)\right]\right]\\
\varphi_2(\mathcal{V},\tau)&=\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left[\frac{\partial^2\varphi_1(\mathcal{V},\tau)}{\partial\mathcal{V}^2}-\mathbb{A}_1(\varphi)+(1+\gamma)\mathbb{B}_1(\varphi)-\gamma\varphi_1(\mathcal{V},\tau)\right]\right]\\
&\vdots\\
\varphi_{m+1}(\mathcal{V},\tau)&=\mathcal{S}^{-1}\left[\frac{v^\delta}{s^\delta}\mathcal{S}\left[\sum_{m=0}^{\infty}\frac{\partial^2\varphi_m(\mathcal{V},\tau)}{\partial\mathcal{V}^2}-\sum_{m=0}^{\infty}\mathbb{A}_m(\varphi)+(1+\gamma)\sum_{m=0}^{\infty}\mathbb{B}_m(\varphi)-\gamma\sum_{m=0}^{\infty}\varphi_m(\mathcal{V},\tau)\right]\right]
\end{aligned}$$

After that, we get the terms

$$\varphi_1(\mathcal{V},\tau) = \frac{(1-2\gamma)\,\tau^{\delta}\ \mathrm{sech}^2\left(\frac{\mathcal{V}}{2\sqrt{2}}\right)}{8\ \Gamma(\delta+1)}$$

$$\varphi_2(\mathcal{V},\tau) = -\frac{(1-2\gamma)^2\,\tau^{2\delta}\ \mathrm{csch}^3\left(\frac{\mathcal{V}}{\sqrt{2}}\right)\Gamma(1+\delta)\ \sinh^4\left(\frac{\mathcal{V}}{2\sqrt{2}}\right)}{2\ \Gamma(\delta+1)\,\Gamma(1+2\delta)}$$

and so forth; The exact solution of FN model give by $\delta = 1$ give by [8, 9, 17]

$$\varphi(\mathcal{V},\tau) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\sqrt{2}\mathcal{V} + (1-2\gamma)\tau}{4}\right)$$

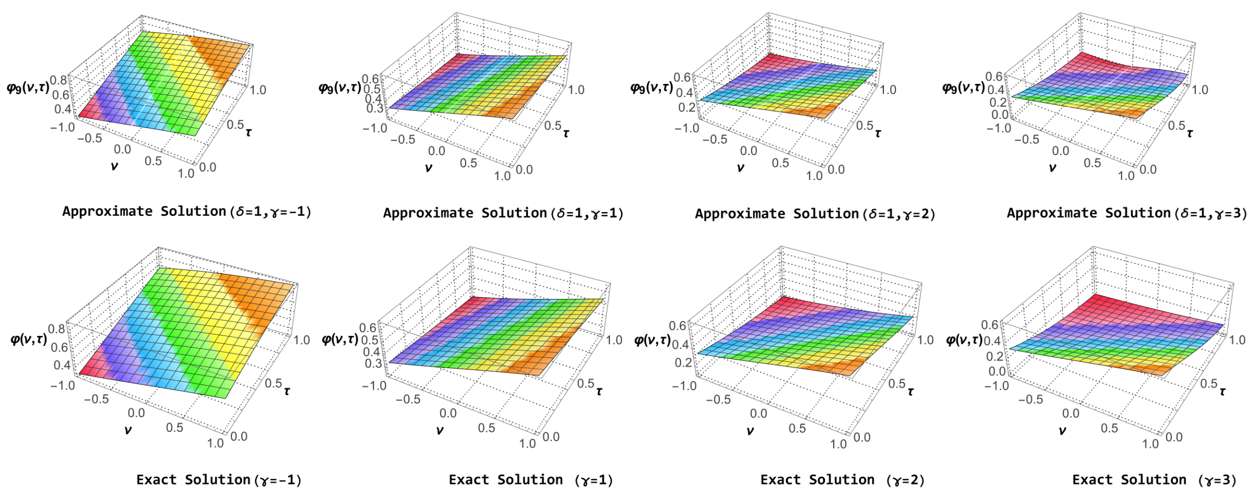

FIGURE 1. Surface of $\varphi_9(\mathcal{V},\tau)$ for different values of γ and exact solution of example 5.1

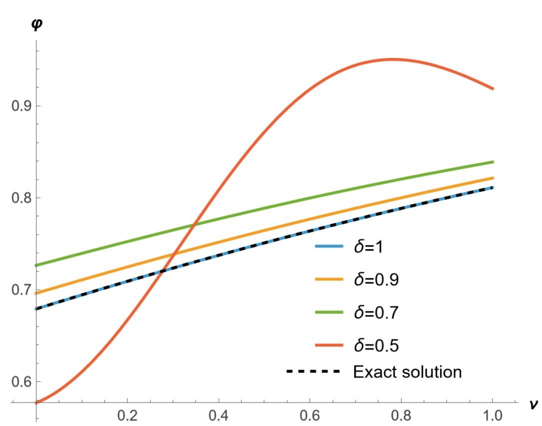

FIGURE 2. $\varphi_9(\mathcal{V},\tau)$ of STADM for different values of δ with the exact solution at $(\tau = 0.5, \gamma = -1)$ of example 5.1.

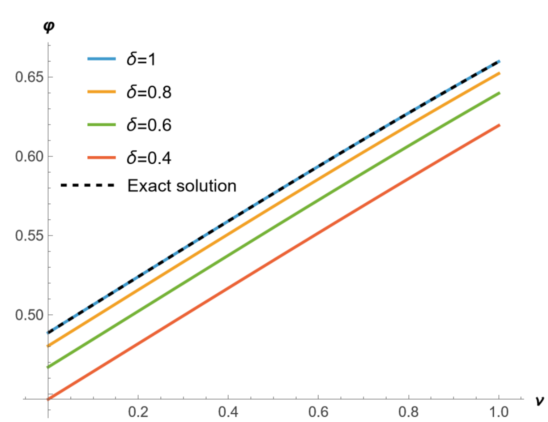

FIGURE 3. $\varphi_9(\mathcal{V},\tau)$ of STADM for different values of δ with the exact solution at $(\tau = 0.09, \gamma = 1)$ of example 5.1.

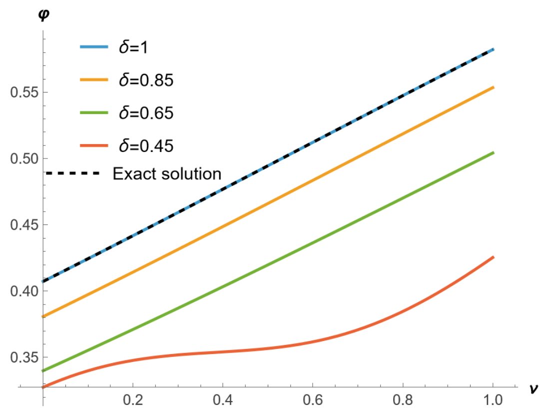

FIGURE 4. $\varphi_9(\mathcal{V},\tau)$ of STADM for different values of δ with the exact solution at $(\tau = 0.25, \gamma = 2)$ of example 5.1.

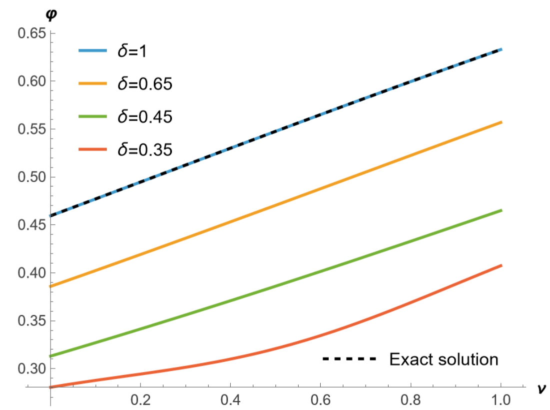

FIGURE 5. $\varphi_9(\mathcal{V},\tau)$ of STADM for different values of δ with the exact solution at $(\tau = 0.065, \gamma = 3)$ of example 5.1.

TABLE 1. Comparison of absolute error for φ_5 and φ_9 at $(\delta = 1,\ \gamma = -1)$.

$(\mathcal{V},\tau)$	Exact	$\varphi_5(\mathcal{V},\tau)$	$\lvert\varphi_5 - \varphi_{Exact}\rvert$	$\varphi_9(\mathcal{V},\tau)$	$\lvert\varphi_9 - \varphi_{Exact}\rvert$
$(0.1, 0.1)$	0.5549547652	0.5549547667	$1.5308E-09$	0.5549547652	$1.1324E-14$
$(0.1, 0.2)$	0.5916306922	0.5916308118	$1.1960E-07$	0.5916306922	$1.3354E-11$
$(0.3, 0.4)$	0.6925636417	0.6925816773	$1.8036E-05$	0.6925636728	$3.1119E-08$
$(0.5, 0.6)$	0.7779143639	0.7781877593	$2.7340E-03$	0.7779164261	$2.0621E-06$
$(0.7, 0.8)$	0.8448772677	0.8464789821	$1.6017E-03$	0.8449062326	$2.8965E-05$
$(0.9, 0.8)$	0.8625219231	0.8640167507	$1.4948E-03$	0.8625390447	$1.7122E-05$
$(0.9, 1.0)$	0.8943906844	0.8997808826	$5.3902E-03$	0.8945282884	$1.3760E-04$

TABLE 2. Comparison of absolute error for φ_5 and φ_9 at $(\delta = 1,\ \gamma = 1)$.

$(\mathcal{V},\tau)$	Exact	$\varphi_5(\mathcal{V},\tau)$	$\lvert\varphi_5 - \varphi_{Exact}\rvert$	$\varphi_9(\mathcal{V},\tau)$	$\lvert\varphi_9 - \varphi_{Exact}\rvert$
$(0.1, 0.1)$	0.5051774845	0.5051774845	$1.4584E-12$	0.5051774845	$0.0000E-00$
$(0.1, 0.2)$	0.4926781929	0.4926781930	$8.2902E-11$	0.4926781929	$1.6653E-16$
$(0.3, 0.4)$	0.5030329714	0.5030329880	$1.6605E-08$	0.5030329714	$1.5434E-11$
$(0.5, 0.6)$	0.5133851488	0.5133854445	$2.9571E-07$	0.5133851488	$5.9824E-10$
$(0.7, 0.8)$	0.5237258551	0.5237279301	$2.0751E-06$	0.5237258557	$6.3013E-10$
$(0.9, 0.8)$	0.5588253353	0.5588276285	$2.2932E-06$	0.5588253358	$5.0934E-10$
$(0.9, 1.0)$	0.5340462594	0.5340549344	$8.6740E-06$	0.5340462642	$4.8331E-09$

TABLE 3. Comparison of absolute error for φ_5 and φ_9 at $(\delta = 1,\ \gamma = 2)$.

$(\mathcal{V},\tau)$	Exact	$\varphi_5(\mathcal{V},\tau)$	$\lvert\varphi_5 - \varphi_{Exact}\rvert$	$\varphi_9(\mathcal{V},\tau)$	$\lvert\varphi_9 - \varphi_{Exact}\rvert$
$(0.1, 0.1)$	0.4801880479	0.4801880487	$8.2508E-10$	0.4801880479	$7.7716E-15$
$(0.1, 0.2)$	0.4429274929	0.4429275228	$2.9871E-08$	0.4429274929	$6.0539E-12$
$(0.3, 0.4)$	0.4042306498	0.4042390590	$8.4092E-06$	0.4042306706	$2.0890E-08$
$(0.5, 0.6)$	0.3666892150	0.3668493599	$1.6015E-04$	0.3666909968	$1.7817E-06$
$(0.7, 0.8)$	0.3306990077	0.3318769198	$1.1779E-03$	0.3307343917	$3.5384E-05$
$(0.9, 0.8)$	0.3627139983	0.3641857275	$1.4717E-03$	0.3627468256	$3.2827E-05$
$(0.9, 1.0)$	0.2965869386	0.3017591132	$5.1722E-03$	0.2968921461	$3.0521E-04$

TABLE 4. Comparison of absolute error for φ_5 and φ_9 at ($\delta = 1,\ \gamma = 3$).

$(\mathcal{V},\tau)$	Exact	$\varphi_5(\mathcal{V},\tau)$	$\lvert\varphi_5-\varphi_{Exact}\rvert$	$\varphi_9(\mathcal{V},\tau)$	$\lvert\varphi_9-\varphi_{Exact}\rvert$
$(0.1, 0.1)$	0.4552973514	0.4552973639	$1.2562E-08$	0.4552973514	$1.0798E-12$
$(0.1, 0.2)$	0.3942960477	0.3942960397	$7.9994E-09$	0.3942960483	$5.8468E-10$
$(0.3, 0.4)$	0.3126266418	0.3127268557	$1.0021E-04$	0.3126293199	$2.6781E-06$
$(0.5, 0.6)$	0.2411387227	0.2433015888	$2.1629E-03$	0.2413796338	$2.4091E-04$
$(0.7, 0.8)$	0.1816772209	0.1984191783	$1.6742E-02$	0.1866661440	$4.9889E-03$
$(0.9, 0.8)$	0.2036551989	0.2281296429	$2.4474E-02$	0.2088767790	$5.2216E-03$
$(0.9, 1.0)$	0.1342835412	0.2102283160	$7.5945E-02$	0.1796222827	$4.5339E-02$

TABLE 5. Comparison of φ_4 by STADM with LRPSM and NIM [9] at ($\gamma = 1, \delta = 1$).

τ	$\mathcal{V}$	$Exact$	$\varphi_4(\mathcal{V},\tau)$	$\lvert\varphi_4-\varphi_{Eaxct}\rvert$	$LRPSM$[9]	NIM [9]
0.1	0.4	0.55794910166	0.55794910221	$5.5138E-10$	$9.2474E-07$	$3.8083E-05$
	0.8	0.62613832216	0.62613832245	$2.9382E-10$	$1.4633E-06$	$7.3756E-05$
	1.2	0.68965954745	0.68965954745	$2.3213E-12$	$1.5086E-06$	$1.0003E-04$
	1.6	0.74675331628	0.74675331607	$2.1220E-10$	$1.1810E-06$	$1.1467E-04$
	2.0	0.79644365864	0.79644365833	$3.0606E-10$	$6.7895E-06$	$1.1803E-04$
0.01	0.4	0.56901725701	0.56901725701	$5.4401E-15$	$9.5102E-10$	$4.0262E-07$
	0.8	0.63661111955	0.63661111955	$2.7756E-15$	$1.4719E-09$	$7.5457E-07$
	1.2	0.69920776181	0.69920776181	$0.0000E-00$	$1.5046E-09$	$1.0108E-06$
	1.6	0.75516853061	0.75516853061	$2.3315E-15$	$1.1705E-09$	$1.1507E-06$
	2.0	0.80364187214	0.80364187214	$3.1086E-15$	$6.6967E-10$	$1.1789E-06$

TABLE 6. Comparison of φ_4 by STADM with LRPSM and NIM [9] at ($\gamma = -0.2, \delta = 1$).

τ	$\mathcal{V}$	$Exact$	$\varphi_4(\mathcal{V},\tau)$	$\lvert\varphi_4-\varphi_{Eaxct}\rvert$	$LRPSM$[9]	NIM [9]
0.1	0.4	0.58730675913	0.58730675624	$2.8885E-09$	$2.5396E-06$	$8.5904E-05$
	0.8	0.65377749489	0.65377749342	$1.4669E-09$	$4.8255E-06$	$1.5325E-04$
	1.2	0.71474210866	0.71474210874	$8.7631E-11$	$6.4549E-06$	$2.0133E-04$
	1.6	0.76876856030	0.76876856150	$1.1984E-09$	$7.2005E-06$	$2.2662E-04$
	2.0	0.81520817101	0.81520817267	$1.6583E-09$	$7.1429E-06$	$2.3044E-04$
0.01	0.4	0.57195762591	0.57195762591	$2.9199E-14$	$2.3629E-09$	$8.0042E-07$
	0.8	0.63938259175	0.63938259175	$1.5321E-14$	$4.6273E-09$	$1.4877E-06$
	1.2	0.70172550816	0.70172550816	$3.3307E-16$	$6.2686E-09$	$1.9866E-06$
	1.6	0.75738039953	0.75738039953	$1.1546E-14$	$7.0502E-09$	$2.2572E-06$
	2.0	0.80552859421	0.80552859421	$1.6542E-14$	$7.0379E-09$	$2.3097E-06$

TABLE 7. Comparison of φ_3 of STADM and HPTT [17] at ($\gamma = -1, \delta = 1$).

$\mathcal{V}$	τ	$Exact$	$\varphi_3(\mathcal{V},\tau)$	$\lvert\varphi_3-\varphi_{Eaxct}\rvert$	$HPTT$ [17]
0.1	0.1	0.5549547652	0.5549542387	$5.2657E-07$	$1.4000E-04$
	0.2	0.5916306922	0.5916198578	$1.0834E-05$	$1.1000E-03$
	0.3	0.6273139316	0.6272473955	$6.6536E-05$	$3.7000E-03$
	0.4	0.6616622737	0.6614170824	$2.4519E-04$	$8.7000E-03$
0.2	0.01	0.5390257991	0.5390257990	$7.5042E-11$	$1.4000E-07$
	0.02	0.5427507035	0.5427507023	$1.2248E-09$	$1.1070E-06$
	0.03	0.5464708338	0.5464708275	$6.3223E-09$	$3.7338E-06$
	0.04	0.5501857817	0.5501857612	$2.0365E-08$	$3.8354E-06$

Example 5.2. Consider the Fitzhugh-Nagumo model for $\gamma = 2$ [5],

$$\mathcal{D}_\tau^\delta \varphi(\mathcal{V},\tau) - \frac{\partial^2 \varphi(\mathcal{V},\tau)}{\partial \mathcal{V}^2} + \varphi^3(\mathcal{V},\tau) - 3\varphi^2(\mathcal{V},\tau) + 2\varphi(\mathcal{V},\tau) = 0, \text{ where } 0 < \delta \leq 1, \tag{5.6}$$

Subject to the initial condition

$$\varphi(\mathcal{V},0) = \frac{e^{\frac{\sqrt{2}\mathcal{V}}{2}} + 2e^{\sqrt{2}\mathcal{V}}}{e^{\frac{\sqrt{2}\mathcal{V}}{2}} + e^{\sqrt{2}\mathcal{V}} + 1} \tag{5.7}$$

Analogous to example 5.1, we obtain the terms

$$\varphi_1(\mathcal{V},\tau) = \frac{3\tau^\delta \sinh\left(\frac{\mathcal{V}}{\sqrt{2}}\right)}{\Gamma(\delta+1)\left(2\cosh\left(\frac{\mathcal{V}}{\sqrt{2}}\right)+1\right)^2}$$

$$\varphi_2(\mathcal{V},\tau) = \frac{9\tau^{2\delta}\left(\sinh\left(\frac{\mathcal{V}}{\sqrt{2}}\right) - \sinh\left(\sqrt{2}\mathcal{V}\right)\right)}{2\Gamma(2\delta+1)\left(2\cosh\left(\frac{\mathcal{V}}{\sqrt{2}}\right)+1\right)^3}$$

and so forth; The exact solution of model (5.6-5.7) when $\delta = 1$ give by [5, 19],

$$\varphi(\mathcal{V},\tau) = \frac{e^{\frac{\sqrt{2}\mathcal{V}}{2} - \frac{3\tau}{2}} + 2e^{\sqrt{2}\mathcal{V}}}{e^{\frac{\sqrt{2}\mathcal{V}}{2} - \frac{3\tau}{2}} + e^{\sqrt{2}\mathcal{V}} + 1} \tag{5.8}$$

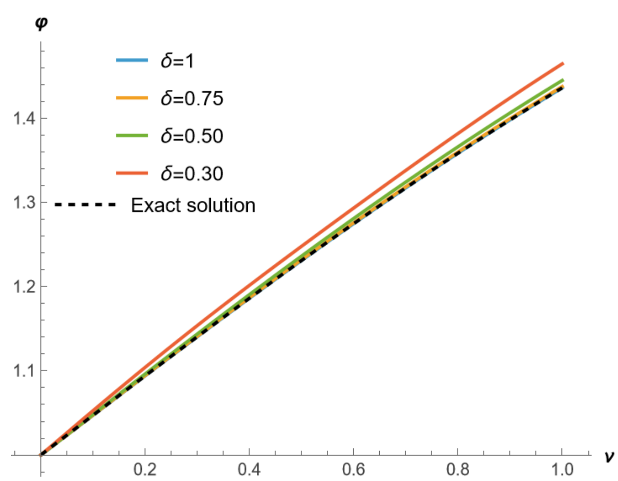

FIGURE 6. $\varphi_{10}(\mathcal{V},\tau)$ of STADM for different values of δ with the exact solution at $\tau = 0.002$ of example 5.2.

TABLE 8. Comparison of absolute error for φ_5 and φ_{10} at $\delta = 1$ of example 5.2.

$(\mathcal{V}, \tau)$	Exact	$\varphi_5(\mathcal{V}, \tau)$	$\lvert\varphi_5 - \varphi_{Exact}\rvert$	$\varphi_{10}(\mathcal{V}, \tau)$	$\lvert\varphi_{10} - \varphi_{Exact}\rvert$
(0.1, 0.2)	1.051547159	1.051547187	$2.8121E-08$	1.051547159	$1.7986E-13$
(0.3, 0.4)	1.164786587	1.164791633	$5.0462E-06$	1.164786586	$1.1008E-09$
(0.5, 0.6)	1.285023894	1.285108520	$8.4626E-05$	1.285023745	$1.4956E-07$
(0.7, 0.8)	1.404066882	1.404614550	$5.4767E-04$	1.404062623	$4.2585E-06$
(0.9, 1.0)	1.514939492	1.516958450	$2.0190E-03$	1.514891660	$4.7831E-05$

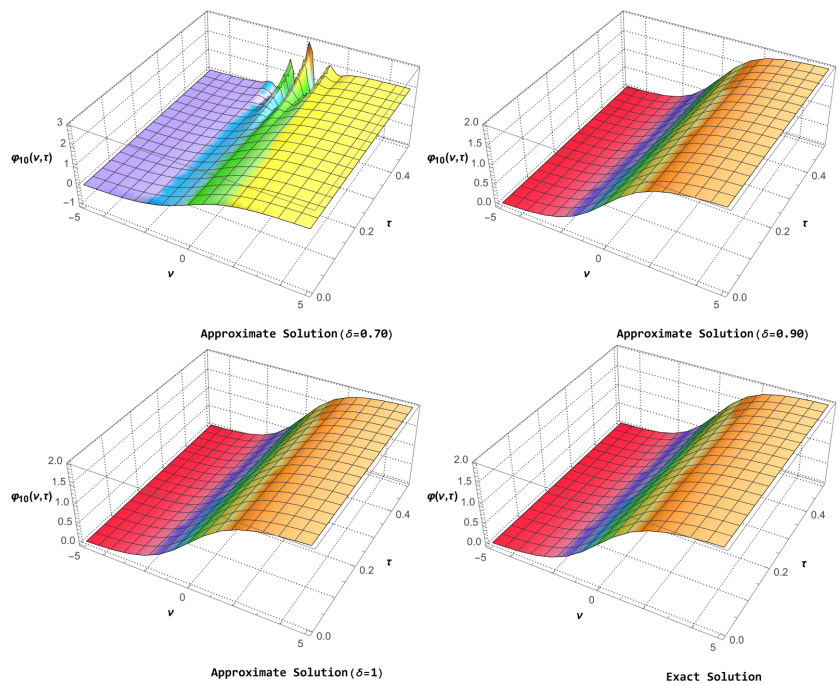

FIGURE 7. Surface of $\varphi_{10}(\mathcal{V}, \tau)$ for different values of δ and exact solution of example 5.2

TABLE 9. Comparison of φ_3 by STADM with MRPSM and MTIM [5] at ($\tau = 0.01, \delta = 1$) of example 5.2.

$\mathcal{V}$	$Exact$	$\varphi_3(\mathcal{V}, \tau)$	$\lvert\varphi_3 - \varphi_{Exact}\rvert$	$MRPSM$ [5]	$MTIM$ [5]
1.0	1.43779716825	1.43779716808	$1.6236E-10$	$1.5163E-08$	$1.1133E-10$
1.1	1.47423277252	1.47423277235	$1.7151E-10$	$1.4171E-08$	$3.7760E-09$
1.2	1.50892956321	1.50892956304	$1.7829E-10$	$1.2737E-08$	$7.9676E-09$
1.3	1.54186265966	1.54186265948	$1.8255E-10$	$1.0917E-08$	$1.2299E-08$
1.4	1.57302724534	1.57302724516	$1.8423E-10$	$8.7748E-08$	$1.6617E-08$
1.5	1.60243629244	1.60243629226	$1.8332E-10$	$6.3809E-08$	$2.0791E-08$
1.6	1.63011814399	1.63011814381	$1.7994E-10$	$3.8081E-08$	$2.4708E-08$
1.7	1.65611406409	1.65611406392	$1.7425E-10$	$1.1284E-08$	$2.8286E-08$
1.8	1.68047584725	1.68047584709	$1.6648E-10$	$1.5895E-08$	$3.1465E-08$
1.9	1.70326355740	1.70326355725	$1.5692E-10$	$4.2829E-08$	$3.4208E-08$
2.0	1.72454344688	1.72454344673	$1.4590E-10$	$6.8963E-08$	$3.6498E-08$

Example 5.3. Consider the Fitzhugh-Nagumo model for $\gamma = 3$ [5],

$$\mathcal{D}_\tau^\delta \varphi(\mathcal{V},\tau) - \frac{\partial^2\varphi(\mathcal{V},\tau)}{\partial\mathcal{V}^2} + \varphi^3(\mathcal{V},\tau) - 4\varphi^2(\mathcal{V},\tau) + 3\varphi(\mathcal{V},\tau) = 0, \text{ where } 0 < \delta \leq 1, \tag{5.9}$$

Subject to the initial condition

$$\varphi(\mathcal{V},0) = \frac{e^{\frac{\sqrt{2}\mathcal{V}}{2}} + 3e^{\frac{3\sqrt{2}\mathcal{V}}{2}}}{e^{\frac{\sqrt{2}\mathcal{V}}{2}} + e^{\frac{3\sqrt{2}\mathcal{V}}{2}} + 1} \tag{5.10}$$

Analogous to example 5.1, we obtain the terms

$$\varphi_1(\mathcal{V},\tau) = \frac{e^{\frac{\mathcal{V}}{\sqrt{2}}}\left(16e^{\frac{3\mathcal{V}}{\sqrt{2}}} + 9e^{\sqrt{2}\mathcal{V}} - 5\right)\tau^\delta}{2\left(e^{\frac{\mathcal{V}}{\sqrt{2}}} + e^{\frac{3\mathcal{V}}{\sqrt{2}}} + 1\right)^2\Gamma(\delta+1)}$$

$$\varphi_2(\mathcal{V},\tau) = -\frac{e^{\frac{\mathcal{V}}{\sqrt{2}}}\tau^{2\delta}\left(e^{\frac{\mathcal{V}}{\sqrt{2}}}\left(-27e^{\frac{\mathcal{V}}{\sqrt{2}}} + e^{\sqrt{2}\mathcal{V}}\left(e^{\sqrt{2}\mathcal{V}}\left(256\sinh\left(\frac{\mathcal{V}}{\sqrt{2}}\right) + 27\right) - 140\right) + 25\right) - 25\right)}{4\left(e^{\frac{\mathcal{V}}{\sqrt{2}}} + e^{\frac{3\mathcal{V}}{\sqrt{2}}} + 1\right)^3\Gamma(2\delta+1)}$$

and so forth; The exact solution of model (5.9-5.10) when $\delta = 1$ give by [5, 19]

$$\varphi(\mathcal{V},\tau) = \frac{e^{\frac{\sqrt{2}\mathcal{V}}{2} - \frac{5\tau}{2}} + 3e^{\frac{3\sqrt{2}\mathcal{V}}{2} + \frac{3\tau}{2}}}{e^{\frac{\sqrt{2}\mathcal{V}}{2} - \frac{5\tau}{2}} + e^{\frac{3\sqrt{2}\mathcal{V}}{2} + \frac{3\tau}{2}} + 1} \tag{5.11}$$

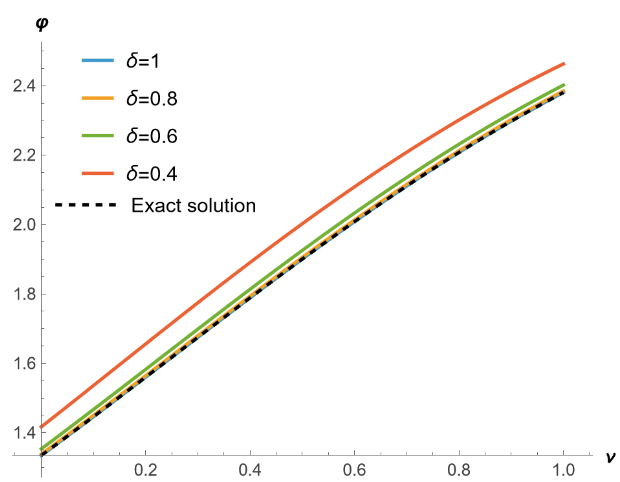

FIGURE 8. $\varphi_8(\mathcal{V},\tau)$ of STADM for different values of δ with the exact solution at $\tau = 0.001$ of example 5.3.

TABLE 10. Comparison of absolute error for φ_4 and φ_8 at $\delta = 1$ of example 5.3.

$(\mathcal{V}, \tau)$	$Exact$	$\varphi_4(\mathcal{V}, \tau)$	$\lvert\varphi_4 - \varphi_{Exact}\rvert$	$\varphi_8(\mathcal{V}, \tau)$	$\lvert\varphi_8 - \varphi_{Exact}\rvert$
$(0.01, 0.02)$	1.367076809	1.367076807	$2.6369E-09$	1.367076809	$1.1102E-15$
$(0.03, 0.04)$	1.413376982	1.413376893	$8.8953E-08$	1.413376982	$2.3981E-13$
$(0.05, 0.06)$	1.461094160	1.461093453	$7.0711E-07$	1.461094160	$9.4449E-12$
$(0.07, 0.08)$	1.510052444	1.510049346	$3.0978E-06$	1.510052443	$1.2751E-10$
$(0.09, 0.10)$	1.560061373	1.560051612	$9.7608E-06$	1.560061372	$9.5202E-10$

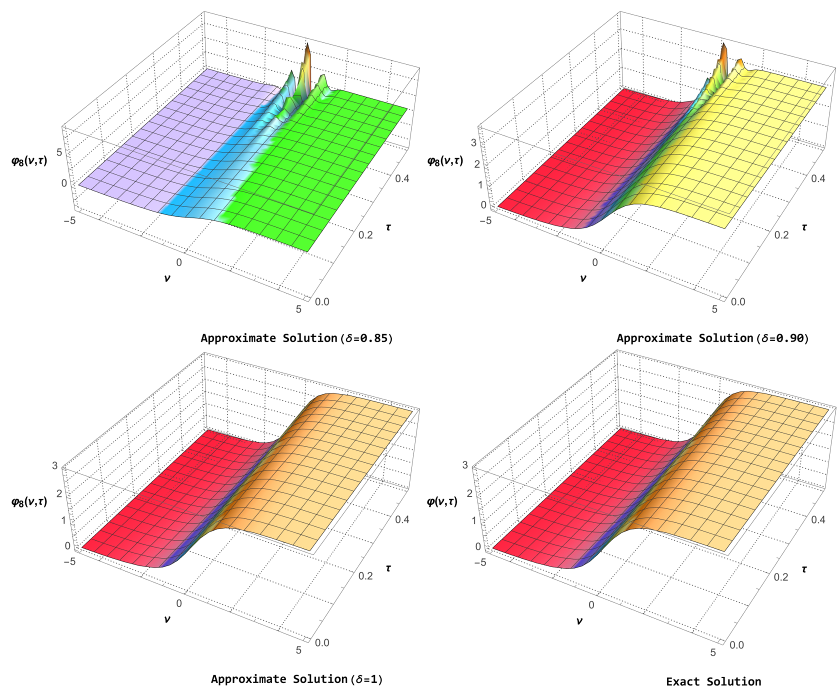

FIGURE 9. Surface of $\varphi_8(\mathcal{V}, \tau)$ for different values of δ and exact solution of example 5.3

TABLE 11. Comparison of φ_4 by STADM with MRPSM and MTIM [5] at $(\tau = 0.01, \delta = 1)$ of example 5.3.

$\mathcal{V}$	$Exact$	$\varphi_4(\mathcal{V}, \tau)$	$\lvert\varphi_4 - \varphi_{Exact}\rvert$	$MRPSM$ [5]	$MTIM$ [5]
1.0	2.39228351940	2.39228351951	$1.0931E-10$	$1.5994E-07$	$1.6082E-06$
1.1	2.46690911161	2.46690911173	$1.2894E-10$	$7.1111E-08$	$1.7230E-06$
1.2	2.53414536028	2.53414536041	$1.3704E-10$	$2.7531E-07$	$1.7712E-06$
1.3	2.59424397939	2.59424397953	$1.3489E-10$	$4.4469E-07$	$1.7629E-06$
1.4	2.64758826193	2.64758826205	$1.2479E-10$	$5.7586E-07$	$1.7104E-06$
1.5	2.69465010524	2.69465010534	$1.0937E-10$	$6.6919E-07$	$1.6259E-06$
1.6	2.73595251671	2.73595251680	$9.1158E-11$	$7.2771E-07$	$1.5204E-06$
1.7	2.77203895460	2.77203895467	$7.2255E-11$	$7.5607E-07$	$1.4029E-06$
1.8	2.80344974475	2.80344974480	$5.4193E-11$	$7.5964E-07$	$1.2807E-06$
1.9	2.83070507185	2.83070507188	$3.7959E-11$	$7.4386E-07$	$1.1591E-06$
2.0	2.85429363268	2.85429363270	$2.4074E-11$	$7.1380E-07$	$1.0414E-06$

6. Discussions and Result

The STADM formulation is tested upon the Fitzhugh-Nagumo model with the Caputo fractional derivative. In figures 1-9 we present the response of the obtained solution by STADM for different values of δ illustrated in both 2D and 3D plots. These plots provide a graphical comparison of the different orders of approximation solutions derived from STADM alongside their exact solutions. In tables 1-11, we present comparisons in the form of absolute errors for the numerical solutions obtained from STADM, HPTT [17], LRPSM [9], NIM [9], MRPSM [5], MTIM [5] and the exact solution. We note that absolute error is largely dropped when modifying the solution by taking further terms; tables 1-4, 8, and 10 explain this fact.

7. Conclusion

In this paper, we have used a combined form of the Adomian decomposition method and the Shehu transform (STADM) to obtain a numerical solution to the Fitzhugh-Nagumo model in the context of the Caputo operator. Our findings demonstrate the suggested method is effective for generating approximate solutions. Additionally, STADM can be considered an efficient and accurate improvement of the existing numerical approaches, and it may be used to explore highly nonlinear partial differential equations that arise in various areas of mathematics.

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