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THE UNIQUENESS OF COUPLED FIXED POINT FOR WEAKLY C-CONTRACTIVE MAPPINGS IN PARTIAL METRIC SPACES

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ABSTRACT. We extend the concept of weakly C- contractive mappings to the context of partial metric spaces. Also, we prove that every weakly C-contractive mapping in partial metric space has a unique coupled fixed point. Our result generalizes several well-known existing results in the literature.

1. INTRODUCTION AND PRELIMINARIES

The most celebrated fixed point theorem is the Banach contraction principle. Afterward many authors obtained various important extensions of this principle(see [13]).

Chatterjea [6] introduced the following contraction which has been named later by C-contraction.

Definition 1.1. [6] Let (X, d) be a metric space and $T : X \to X$ be a mapping. Then T is called a C-contraction if there exists $k \in [0, \frac{1}{2})$ such that

$$d(T(x), T(y)) \leq k(d(x, T(y)) + d(T(x), y))$$

holds for all $x, y \in X$.

Under this kind of contraction, Chatterjea [6] established the following fixed point result:

Theorem 1.2. [6] *Every C-contraction in a complete metric space has a unique fixed point.*

As a generalization of C-contractive mapping, Choudhury [7] introduced the concept of weakly C-contractive mapping and proved that every weakly C-contractive mapping in a complete metric space has a unique fixed point.

Definition 1.3. [7] Let (X, d) be a metric space and $T : X \to X$ be a mapping. Then T is called a weakly C-contractive if T satisfies that

$$d(T(x), T(y)) \leq \frac{1}{2}(d(x, T(y)) + d(T(x), y)) - \phi\left(d(x, T(y)), d(T(x), y)\right))$$

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for all $x, y \in X$, where $\phi : [0,\infty) \times [0,\infty) \to [0,\infty)$ is a continuous mapping such that $\phi(t, s) = 0$ if and only if $t = s = 0$.

Under this kind of contraction, Choudhury [7] established the following fixed point result:

Theorem 1.4 ([7], Theorem2.1)**.** *Every weakly C-contraction in a complete metric space has a unique fixed point.*

Recently, Harjani et al. [10] studied some fixed point results for weakly C-contractive mappings in a complete metric space endowed with a partial order. Moreover, Shatanawi [17] proved some fixed point and coupled fixed point theorems for a nonlinear weakly C-contraction type mapping in metric and ordered metric spaces.

In another aspect, the notion of a partial metric space has been introduced by Matthews [15] in 1994 as a generalization of the usual metric in such a way that each object doesnt necessarily have to have a zero distance from itself. A motivation behind introducing the concept of a partial metric was to obtain appropriate mathematical models in the theory of computation and, in particular, to give a modified version of the Banach contraction principle (see for example, [8, 18]). Subsequently, several authors studied the problem of existence and uniqueness of a fixed point for mappings satisfying different contractive conditions on partial metric spaces (e.g.,[3, 4, 12, 9, 16]).

We recall some definitions and properties of partial metric spaces.

Definition 1.5. A *partial metric* on a nonempty set X is a function $p : X \times X \longrightarrow \mathbb{R}^+$ such that for all $x, y, z \in X$,

(p1.) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$.
(p2.) $p(x, x) \leq p(x, y)$.
(p3.) $p(x, y) = p(y, x)$.
(p4.) $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$.

A partial metric space is a pair (X, p) such that X is nonempty set and p is a partial metric on X.

From the above definition, if $p(x, y) = 0$, then $x = y$. But if $x = y$, $p(x, y)$ may not be 0 in general. A famous example of a partial metric space is the pair $(\mathbb{R}^+, p)$, where $p : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is defined as $p(x, y) = \max\{x, y\}$. For some more examples of partial metric spaces, we refer to [18, 9].

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls $\{B_p(x, \epsilon) : x \in X, \quad \epsilon > 0\}$ where $B_p(x, \epsilon) = \{y \in X : p(x, y) < p(x, x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$. A sequence $\{x_n\}$ in X converges to a point $x \in X$, with respect to τ_p if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$. A sequence $\{x_n\}$ in X is called Cauchy sequence if $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite.

Definition 1.6. [15, 16] Let be (X, p) a partial metric space. Then,

(i) a sequence $\{x_n\}$ in a partial metric space (X, p) converges to a point $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$;

(ii) a sequence $\{x_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n,x_m)$;
(iii) a partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$, that is, $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

If p is a partial metric on X , then the function $p^s : X \times X \longrightarrow \mathbb{R}^+$ given by

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y),$$

is a metric on X.

Lemma 1.7. [15, 16] *Let (X,p) be a partial metric space. Then*

(a) $\{x_n\}$ is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s).

(b) (X,p) is complete if and only if the metric space (X,p^s) is complete. Furthermore, $\lim_{n\to\infty} p^s(x_n,x) = 0$ if and only if

$$p(x,x) = \lim_{n\to\infty} p(x_n,x) = \lim_{n,m\to\infty} p(x_n,x_m).$$

Moreover, Bhaskar and Lakshmikantham [5] presented coupled fixed point theorems for contractions in partially ordered metric spaces. For more related work on coupled fixed and coincidence points results we refer the readers to recent work in [1]–[5], [10]–[11].

Definition 1.8. [14] Let $F : X \times X \to X$ be a mapping. We say that $(x,y) \in X \times X$ is a coupled fixed point of F if $F(x,y) = x$ and $F(y,x) = y$.

In this paper, we extend the concept of a weakly C-contractive mapping to the context of partial metric space and we prove that every weakly C-contractive mapping in a complete partial metric space has a unique coupled fixed point. Our result generalizes several well known result in the literature.

2. Unique Coupled Fixed Point Theorem

Definition 2.1. Let (X,p) be a partial metric space. Then the mapping $F : X \times X \to X$ is said to be weakly C-contractive if

$$p(F(x,y),F(u,v)) \le \frac{1}{2}(p(x,F(u,v))+p(F(x,y),u)) - \phi\left(p(x,F(u,v)),p(F(x,y),u)\right) \tag{2.1}$$

for all $x,y,u,v \in X$, where $\phi : [0,\infty)\times[0,\infty) \to [0,\infty)$ is a continuous mapping such that $\phi(t,s) = 0$ if and only if $t = s = 0$.

Now we state and prove our main result.

Theorem 2.2. *Let (X,p) be a complete partial metric space and $F : X\times X \to X$ be a weakly C-contraction mapping. Then F has a unique coupled fixed point in X.*

Proof. Let $x_0, y_0 \in X$ be arbitrary points in X. We construct sequences $\{x_n\}$ and $\{y_n\}$ in X as

$$x_{n+1} = F(x_n,y_n) \quad \text{and} \quad y_{n+1} = F(y_n,x_n) \ \text{ for all } n \ge 0. \tag{2.2}$$

Set $\delta_n = p(x_n, x_{n+1}) + p(y_n, y_{n+1})$.

If there exists $n \in \mathbb{N}$ such that $\delta_n = 0$, then by $(p1)$ and $(p2)$ we have $x_n = x_{n+1} = F(x_n, y_n)$ and $y_n = y_{n+1} = F(y_n, x_n)$. Hence, F has a coupled fixed point in X. Now assume that $\delta_n \neq 0$ for all $n \geq 0$.Thus by (2.1), we have

$$\begin{aligned}
p(x_{n+1}, x_{n+2}) &= p(F(x_n, y_n), F(x_{n+1}, y_{n+1})) \\
&\leq \frac{1}{2}(p(x_n, F(x_{n+1}, y_{n+1})) + p(F(x_n, y_n), x_{n+1})) \\
&- \phi\left(p(x_n, F(x_{n+1}, y_{n+1})), p(F(x_n, y_n), x_{n+1})\right) \\
&= \frac{1}{2}(p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})) \\
&- \phi\left(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\right). \qquad (2.3)
\end{aligned}$$

By property $(p4)$, we have

$$p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1}) \leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}).$$

Thus from (2.3), we have

$$\begin{aligned}
p(x_{n+1}, x_{n+2}) &\leq \frac{1}{2}(p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})) \\
&- \phi\left(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\right) \qquad (2.4) \\
&\leq \frac{1}{2}(p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2})) \\
&- \phi\left(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\right) \\
&\leq \max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2})\} \\
&- \phi\left(p(x_n, x_{n+2}), p(x_{n+1}, x_{n+1})\right). \qquad (2.5)
\end{aligned}$$

Similarly,

$$\begin{aligned}
p(y_{n+1}, y_{n+2}) &\leq \frac{1}{2}(p(y_n, y_{n+2}) + p(y_{n+1}, y_{n+1})) \\
&- \phi\left(p(y_n, y_{n+2}), p(y_{n+1}, y_{n+1})\right) \qquad (2.6) \\
&\leq \frac{1}{2}(p(y_n, y_{n+1}) + p(y_{n+1}, y_{n+2})) \\
&- \phi\left(p(y_n, y_{n+2}), p(y_{n+1}, y_{n+1})\right) \\
&\leq \max\{p(y_n, y_{n+1}), p(y_{n+1}, y_{n+2})\} \\
&- \phi\left(p(y_n, y_{n+2}), p(y_{n+1}, y_{n+1})\right). \qquad (2.7)
\end{aligned}$$

Thus we have

$$\begin{aligned}
p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2}) &\leq \frac{1}{2}(p(x_n,x_{n+2})+p(y_n,y_{n+2})+p(x_{n+1},x_{n+1})+p(y_{n+1},y_{n+1}))\\
&- \phi\left(p(x_n,x_{n+2}),p(x_{n+1},x_{n+1})\right)\\
&- \phi\left(p(y_n,y_{n+2}),p(y_{n+1},y_{n+1})\right) \qquad (2.8)\\
&\leq \max\{p(x_n,x_{n+1})+p(y_n,y_{n+1}),p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2})\}\\
&- \phi\left(p(x_n,x_{n+2}),p(x_{n+1},x_{n+1})\right)\\
&- \phi\left(p(y_n,y_{n+2}),p(y_{n+1},y_{n+1})\right). \qquad (2.9)
\end{aligned}$$

From (2.8), we have either $p(x_n,x_{n+2})\neq 0$ or $p(x_{n+1},x_{n+1})\neq 0$ and either $p(y_n,y_{n+2})\neq 0$ or $p(y_{n+1},y_{n+1})\neq 0$ since $p(x_n,x_{n+1})+p(y_n,y_{n+1})\neq 0$ for all $n\in\mathbb{N}$.

If

$$\max\{p(x_n,x_{n+1})+p(y_n,y_{n+1}),p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2})\}=p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2}),$$

then since $\phi\left(p(x_n,x_{n+2}),p(x_{n+1},x_{n+1})\right)>0$ and by (2.9), we have

$$\begin{gathered}
p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2})\leq p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2})\\
-\phi\left(p(x_n,x_{n+2}),p(x_{n+1},x_{n+1})\right)-\phi\left(p(y_n,y_{n+2}),p(y_{n+1},y_{n+1})\right)\\
<p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2}).
\end{gathered}$$

which is a contradiction . Thus, we have

$$\max\{p(x_n,x_{n+1})+p(y_n,y_{n+1}),p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2})\}=p(x_n,x_{n+1})+p(y_n,y_{n+1})$$

and therefore

$$\begin{gathered}
p(x_{n+1},x_{n+2})+p(y_{n+1},y_{n+2})\leq p(x_n,x_{n+1})+p(y_n,y_{n+1})\\
-\phi\left(p(x_n,x_{n+2}),p(x_{n+1},x_{n+1})\right)-\phi\left(p(y_n,y_{n+2}),p(y_{n+1},y_{n+1})\right)\\
\leq p(x_n,x_{n+1})+p(y_n,y_{n+1}).
\end{gathered}$$

By the above inequalities, we have that $\{\delta_n\}=\{p(x_n,x_{n+1})+p(y_n,y_{n+1})\}$ is a non-increasing sequence of positive real numbers. Therefore, there is some $\delta\geq 0$ such that

$$\lim_{n\to\infty}p(x_n,x_{n+1})+p(y_n,y_{n+1})=\delta. \qquad (2.10)$$

Then taking the limit as $n\to\infty$ in (2.9) we have

$$\delta\leq\delta-\lim_{n\to\infty}\phi\left(p(x_n,x_{n+2}),p(x_{n+1},x_{n+1})\right)-\lim_{n\to\infty}\phi\left(p(y_n,y_{n+2}),p(y_{n+1},y_{n+1})\right)\leq\delta.$$

Then

$$\delta-\lim_{n\to\infty}\phi\left(p(x_n,x_{n+2}),p(x_{n+1},x_{n+1})\right)-\lim_{n\to\infty}\phi\left(p(y_n,y_{n+2}),p(y_{n+1},y_{n+1})\right)=\delta.$$

and therefore

$$\lim_{n\to\infty}\phi\left(p(x_n,x_{n+2}),p(x_{n+1},x_{n+1})\right)+\lim_{n\to\infty}\phi\left(p(y_n,y_{n+2}),p(y_{n+1},y_{n+1})\right)=0.$$

By continuity of ϕ, we conclude that

$$\lim_{n\to\infty}p(x_n,x_{n+2})=0, \qquad (2.11)$$

$$\lim_{n\to\infty} p(x_{n+1}, x_{n+1}) = 0, \tag{2.12}$$

$$\lim_{n\to\infty} p(y_n, y_{n+2}) = 0, \tag{2.13}$$

and

$$\lim_{n\to\infty} p(y_{n+1}, y_{n+1}) = 0. \tag{2.14}$$

Letting $n \to \infty$ in (2.8) and (2.10), (2.11), (2.12) and the continuity of ϕ, we conclude that $\delta = 0$. Thus

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = 0, \tag{2.15}$$

and

$$\lim_{n\to\infty} p(y_n, y_{n+1}) = 0. \tag{2.16}$$

Next, We shall prove that

$$\lim_{n,m\to\infty} p(x_n, x_m) = 0.$$

Suppose the contrary, that is

$$\lim_{n,m\to\infty} p(x_n, x_m) \neq 0.$$

Then there exists an $\epsilon > 0$ for which we can find subsequences $\{x_{n(k)}\}, \{x_{m(k)}\}$ of $\{x_n\}$ such that $n(k)$ is the smallest integer for which

$$n(k) > m(k) \geq k, p(x_{n(k)}, x_{m(k)}) \geq \epsilon.$$

This means that

$$p(x_{n(k)-1}, x_{m(k)}) < \epsilon.$$

From the above two inequalities and $(p4)$, we have

$$\begin{aligned}
\epsilon \leq p(x_{n(k)}, x_{m(k)}) & \\
&\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)}) - p(x_{n(k)-1}, x_{n(k)-1}) \\
&\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)}) \\
&< \epsilon + p(x_{n(k)}, x_{n(k)-1}).
\end{aligned}$$

Letting $k \to \infty$ and using (2.15), we get

$$\lim_{k\to\infty} p(x_{n(k)}, x_{m(k)}) = \epsilon. \tag{2.17}$$

By $(p3)$ and $(p4)$, we have

$$\begin{aligned}
&p(x_{n(k)},x_{m(k)})\\
&\leq p(x_{n(k)},x_{n(k)+1})+p(x_{n(k)+1},x_{m(k)})-p(x_{n(k)+1},x_{n(k)+1})\\
&\leq p(x_{n(k)},x_{n(k)+1})+p(x_{n(k)+1},x_{m(k)})\\
&\leq p(x_{n(k)},x_{n(k)+1})+p(x_{n(k)+1},x_{m(k)+1})+p(x_{m(k)+1},x_{m(k)})-p(x_{m(k)+1},x_{m(k)+1})\\
&\leq p(x_{n(k)},x_{n(k)+1})+p(x_{n(k)+1},x_{m(k)+1})+p(x_{m(k)+1},x_{m(k)})\\
&\leq 2p(x_{n(k)},x_{n(k)+1})+p(x_{n(k)},x_{m(k)+1})+p(x_{m(k)+1},x_{m(k)})-p(x_{n(k)},x_{n(k)})\\
&\leq 2p(x_{n(k)},x_{n(k)+1})+p(x_{n(k)},x_{m(k)+1})+p(x_{m(k)+1},x_{m(k)})\\
&\leq 2p(x_{n(k)},x_{n(k)+1})+p(x_{n(k)},x_{m(k)})+2p(x_{m(k)+1},x_{m(k)})-p(x_{m(k)},x_{m(k)})\\
&\leq 2p(x_{n(k)},x_{n(k)+1})+p(x_{n(k)},x_{m(k)})+2p(x_{m(k)+1},x_{m(k)})
\end{aligned}$$

Letting $k\to+\infty$ in the above inequalities and using (2.15),(2.17), we have

$$\begin{aligned}
\lim_{k\to+\infty}p(x_{n(k)},x_{m(k)}) &= \lim_{k\to+\infty}p(x_{n(k)+1},x_{m(k)})\\
&= \lim_{k\to+\infty}p(x_{n(k)+1},x_{m(k)+1})\\
&= \lim_{k\to+\infty}p(x_{n(k)},x_{m(k)+1})=\epsilon. \qquad (2.18)
\end{aligned}$$

Therefore, from (2.1), we have

$$\begin{aligned}
&p(x_{m(k)+1},x_{n(k)+1})\\
&= p(F(x_{m(k)},x_{m(k)}),F(x_{n(k)},x_{n(k)}))\\
&\leq \frac{1}{2}(p(x_{m(k)},F(x_{n(k)},x_{n(k)}))+p(F(x_{m(k)},x_{m(k)}),x_{n(k)}))\\
&-\phi\ \big(p(x_{m(k)},F(x_{n(k)},x_{n(k)})),p(F(x_{m(k)},x_{m(k)}),x_{n(k)}))\big)\\
&= \frac{1}{2}(p(x_{m(k)},x_{n(k)+1})+p(x_{m(k)+1},x_{n(k)}))\\
&- \phi\left(p(x_{m(k)},x_{n(k)+1}),p(x_{m(k)+1},x_{n(k)}))\right)
\end{aligned}$$

Letting $k\to+\infty$ in the above and using (2.18) and the continuity of ϕ, we conclude that

$$\epsilon\leq\epsilon-\phi(\epsilon,\epsilon)<\epsilon$$

which is a contradiction. Thus, we have

$$\lim_{n,m\to+\infty}p(x_n,x_m)=0.$$

Therefore $\{x_n\}$ is a Cauchy sequence in the complete metric space (X,p). By Lemma1.7, we have that

$$\lim_{n,m\to+\infty}p^s(x_n,x_m)=0.$$

Thus, $\{x_n\}$ is a Cauchy sequence in the complete metric space (X, p^s). Hence, by Lemma 1.7,$\{x_n\}$ is a Cauchy sequence in the complete metric space (X, p). Again, by Lemma 1.7, there exists $x \in X$ such that

$$\lim_{n\to+\infty} p^s(x_n, x) = 0$$

if and only if

$$p(x, x) = \lim_{n\to+\infty} p(x_n, x) = \lim_{n,m\to+\infty} p(x_n, x_m). \tag{2.19}$$

Similarly, we can show that $\{y_n\}$ is a Cauchy sequence in the complete metric space (X, p) and there exists $y \in X$ such that

$$\lim_{n\to+\infty} p^s(y_n, y) = 0$$

if and only if

$$p(y, y) = \lim_{n\to+\infty} p(y_n, y) = \lim_{n,m\to+\infty} p(y_n, y_m).$$

Next, we prove that $\lim_{n\to+\infty} p(F(x, y), x_{n+1}) = p(F(x, y), x)$.

Letting $n \to +\infty$ in

$$p(F(x, y), x_{n+1}) \le p(F(x, y), x) + p(x, x_{n+1}) - p(x, x),$$

we have

$$\lim_{n\to+\infty} p(F(x, y), x_{n+1}) \le p(F(x, y), x) \tag{2.20}$$

Also, letting $n \to +\infty$ in

$$p(F(x, y), x) \le p(F(x, y), x_{n+1}) + p(x_{n+1}, x) - p(x_{n+1}, x_{n+1}),$$

we have

$$p(F(x, y), x) \le \lim_{n\to+\infty} p(F(x, y), x_{n+1}) \tag{2.21}$$

From (2.20) and (2.21) , we have

$$\lim_{n\to+\infty} p(F(x, y), x_{n+1}) = p(F(x, y), x) \tag{2.22}$$

Now, we prove that $F(x, y) = x$. By (2.1), we have

$$\begin{aligned}
p(F(x, y), x_{n+1}) &= p(F(x, y), F(x_n, y_n)) \\
&\le \frac{1}{2}(p(x, F(x_n, y_n)) + p(F(x, y), x_n)) \\
&- \phi\left(p(x, F(x_n, y_n)), p(F(x, y), x_n)\right) \\
&= \frac{1}{2}(p(x, x_{n+1}) + p(F(x, y), x_n)) \\
&- \phi\left(p(x, x_{n+1}), p(F(x, y), x_n)\right).
\end{aligned}$$

Letting $n \to +\infty$ in the above and using (2.22) and the continuity of ϕ, we conclude that

$$p(F(x, y), x) \le \frac{1}{2}p(F(x, y), x) - \phi\left(0, p(F(x, y), x)\right) \le \frac{1}{2}p(F(x, y), x).$$

Hence $p(F(x, y), x) = 0$. By $(p1)$ and $(p2)$,we have $F(x, y) = x$. Similarly we can show that $F(y, x) = y$.

Thus, (x, y) is a coupled fixed point of F .

To prove the uniqueness of the coupled fixed point. Suppose that (u,v) is another coupled fixed point of F. From (2.1), we have

$$p(x,u)=p(F(x,y),F(u,v))\leq \frac{1}{2}(p(x,u)+p(x,u))-\phi\left(p(x,u),p(x,u)\right).$$

Therefore, we have $\phi\left(p(x,u),p(x,u)\right)=0$. Hence, $p(x,u)=0$. By $(p1)$ and $(p2)$, we have $x=u$. Similarly, we can show that $y=v$.
Thus F has a unique coupled fixed point.

□

As an immediate consequence of the above theorem, we have the following fixed point result:

Corollary 2.3. *Let (X,p) be a complete partial metric space and $T:X\to X$ be a weakly C-contraction mapping.i.e.T satisfies that*

$$p(T(x),T(y))\leq \frac{1}{2}(p(x,T(y))+p(T(x),y))-\phi\left(p(x,T(y)),p(T(x),y)\right)$$

for all $x,y\in X$, where $\phi:[0,\infty)\times[0,\infty)\to[0,\infty)$ is a continuous mapping such that $\phi(t,s)=0$ if and only if $t=s==0$.
Then there exist a unique $x\in X$ such that $T(x)=x$.

Moreover, Theorem 2.2 is a generalization of Aydi's results in Theorem 2.5 and Corollary 2.7 [4].

Corollary 2.4 ([4], Theorem 2.5)**.** *Let (X,p) be a complete partial metric space. Suppose that the mapping $F:X\times X\to X$ satisfies the following contractive condition*

$$p(F(x,y),F(u,v))\leq\ kp(F(x,y),u)+lp(x,F(u,v)),$$

for all $x,y,u,v\in X$, where k,l are nonnegative constants with $k+2l<1$. Then F has a unique coupled fixed point.

Proof. Take $\phi(t,s)=(\frac{1}{2}-l)t+(\frac{1}{2}-k)s$ where k,l are nonnegative constants with $k+2l<1$. □

Corollary 2.5 ([4], Corollary 2.7)**.** *Let (X,p) be a complete partial metric space. Suppose that the mapping $F:X\times X\to X$ satisfies the following contractive condition*

$$p(F(x,y),F(u,v))\leq\ \frac{k}{2}(p(F(x,y),u)+p(x,F(u,v))),$$

for all $x,y,u,v\in X$, where $0\leq k<\frac{2}{3}$. Then F has a unique coupled fixed point.

Proof. Take $\phi(t,s)=(\frac{1}{2}-\frac{k}{2})(t+s)$ where $0\leq k<\frac{2}{3}$. □

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