

MISCONCEPTIONS OF MATHEMATICAL CONCEPTS VIS-À-VIS HOW THEY POSE AS BARRIERS TO DEVELOPING STUDENTS CONCEPTUAL UNDERSTANDING

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ABSTRACT. This paper is a report of a qualitative case study conducted in South Africa that sought to identify students' misconceptions and how those misconceptions hinder students' conceptual understanding of mathematical concepts, with a focus on algebraic expressions. The Structure of the Observed Learning Outcomes (SOLO) theory was adopted as the theoretical framework. The media for data collection were: classroom observations, activities, unstructured interviews and standardized tests. Participants were required to explain their solutions to the standardized test via unstructured interviews. This guided the researchers to identify students' misconceptions and how they hindered their conceptual understanding of mathematical concepts. Content analysis was carried out on participants' written and oral responses by applying the SOLO theory. From this study, the researchers established that conceptual misunderstandings are the main students' misconception in algebraic expressions. This misconception serves as barriers to students developing conceptual understanding as it does not invoke students' rational reasoning.

1. INTRODUCTION AND PRELIMINARIES

Developing students' conceptual understanding of mathematical concepts is paramount in attaining mathematical proficiency [3, 5, 4]. However, assumptions, preconceptions and misconceptions of mathematical theories and concepts obstruct students' understanding [24]. These develop into errors in students' work, leading to students experiencing severe learning difficulties; this is displayed in their written responses in examinations making them under-achieve in the subject. Recurring misconceptions leads to improper generalisations which deter students' conceptualization and assimilation of mathematical concepts. These in turn impede students from nurturing the relevant mental structures to elicit the appropriate reflective abstractions needed for reflective, logical and advanced mathematical thinking [3] thereby, evoking cognitive conflicts [4].

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This study investigated students' misconceptions in relation to one of the fundamental and challenging mathematics content areas - algebra, with a focus on algebraic expressions [30]. According to [31] cited in [23] algebra encompasses varied perspectives such as: a generalized arithmetic; a problem-solving tool; the study of relationships and a study of structures. [23] avers that "Success in algebra is widely regarded as the "gatekeeper" to success in high school and upper-level mathematics"; the author continues that there is dire need to "build students' conceptual understanding and reasoning in algebra". The instructional objectives and/or aims of algebraic expressions for Grade 10 demand that students are able to - multiply a binomial by a trinomial; factorize trinomials; factorize the difference and sums of two cubes; factorize by grouping in pairs and simplifying, to add and subtract algebraic fractions with denominators involving sum and difference of cubes [9].

According to [26], although errors and misconceptions are similar, there is a need to acknowledge that they are not the same. Errors originate from misconceptions and according to [30] "a misconception is typically classified as flawed understanding of a concept causing repeated errors". [12] advise that if students' misconceptions, rather than errors are identified and corrected it will optimize their conceptual understanding. This explains why it is relevant to focus on students' misconceptions - the primary aim of this study. A myriad of misconceptions leads to ideation of perceived multiple meanings, erroneous conceptions and perceptions of a particular concept [7]. This in-turn become a preconception of a higher concept which required the latter concept as prior knowledge, however, [6] argue that not all preconceptions are misconceptions and misconceptions are barriers to conceptual understanding.

[25] notes that "misconceptions are often deeply held, largely unexplained, and sometimes strongly defended". This imply that misconceptions, when assimilated by students become difficult to be corrected and they lead to misapplications during problem-solving. [10] maintains that "because students have actively constructed their misconceptions from their experiences, they are very attached to them. They find it very difficult to give them up". Is this to say that some students who hold unsubstantiated assumptions leading them to develop misconceptions of a mathematical concept, is an indication of low intelligence and lack of profundity? [22], however, aver that "students with high mathematical creative thinking also experience misconceptions in solving mathematical problems". This presupposes that high intelligent students can as well develop misconceptions of mathematical concepts.

This study required the implementation of an analytic and assessment tool which

guided the researchers to identify students' misconceptions and how they interfered with their conceptual understanding of algebraic expressions. In realising this aim, the knowledge structures of the SOLO model were partly applied: pre-structure level, uni-structure level, multi-structure level, relational level, and extended abstract level [26]. This model accorded the researchers the leeway to monitor and assess students' intellectual ability entailing - reasoning, rate of learning, abstract thinking and making connections between ideas [2].

A plethora of research on students' misconceptions in relation to algebra is available in literature [24]. However, [13] accentuated that most of the studies conducted focused on students' misconceptions instead of how they can be managed. Furthermore, how misconceptions pose as barriers to developing students' conceptual understanding of mathematical concepts, specifically, is limited in literature; this is what the researchers sought to address in this study. In fulfilment of this intention, we aim to answer these critical research questions: (1) Which misconceptions are participants faced with as they learn and solve algebraic expressions problems? (2) How do the identified misconceptions hinder participants' conceptual understanding of algebraic expressions?

2. THEORETICAL FRAMEWORK

The Structure of the Observed Learning Outcomes (SOLO) model was adopted as the theoretical framework for this study as it is an approach for assessing the cognitive and developmental growth of students both quantitatively and qualitatively. The model is a build-up of Piaget's cognitive stage theory[11]. The SOLO model was originally partitioned into five hierarchical abstraction/complexity levels of learning/thinking structures; they are each presented and elaborated in Table 1 [19, 2]. This model, however, has been revised by proponents and other researchers [11]. For instance, the SOLO model was revised to encompass only three structures - the first and last levels (pre-structure level and extended abstract levels) were taking out of the original SOLO model [11]. This meant that this revised SOLO model entailed three thinking levels: one structure, multi-structures and relational structures. The revised SOLO model was applied in this study to qualitatively determine participants' learning outcomes in algebraic expressions.

3. METHODOLOGY

3.1. Research design.

In this study, a case study was implemented as the research design. This guided the researchers to obtain comprehensive occurrences of activities and events at the research field over an extended period of time with particular focus on identifying participants' misconceptions in algebraic expressions and how they become

barriers to participants' conceptual understanding.

3.2. Development of instrument- standardized test.

Different degree of questions based on the thinking structures were put together to constitute the standardized test. The items of this standardized test consisting of five questions were scaffolded: uni-structure level (Questions 1 and 2), multi-structure level (Questions 3 and 4) and relational level (Question 5). A description of how the standardized test instrument was designed to elicit levels from the SOLO taxonomy is presented in Table 1.

3.3. Participants.

The study was carried out at a high school in the Frances Baard Education District under the Northern Cape Department of Education in South Africa. The researchers purposefully located this school where this case study was conducted. The subjects in this study were 86 Grade 10 learners, spread across three classes: 10 A ($n=26$); 10 B ($n=22$) and 10 C ($n=38$); they were all taught by the same teacher at the same school. All these learners consented to partake in this study, thus, they served as subjects without regard to their gender, ethnicity, social and race backgrounds.

3.4. Data analysis.

In this analysis, the SOLO model was applied to determine participants' cognitive development based on their written responses to algebraic expressions, from which participants' levels of thinking with reference to the thinking structures of the SOLO model were examined. In categorizing participants' levels of thinking, one idea connoted one correct response -rated as uni-structure and many ideas connoted many, but not all, correct responses - rated as multi-structures [19]. This analysis was conducted in two stages. Stage one encompassed the presentation of participants' actual written responses; participants were also requested to explain their solutions to the standardized test through an unstructured interview. From these procedures, participants' misconceptions were identified, justified and expounded. Stage two entailed presentation of results after participants' written responses were assessed by applying the SOLO model.

3.5. Stage 1.

At this point participants' actual written responses to sampled tasks from which misconceptions were located are presented. Essential to identifying participants' misconceptions was that the researchers conduct an unstructured interview; this was used as a medium to get participants to justify their written responses. Out of the 86 participants, 10 participants were randomly selected to serve as interviewees. The interview data was captured via audio recordings of participants' oral responses. During the unstructured interviews, two critical questions were

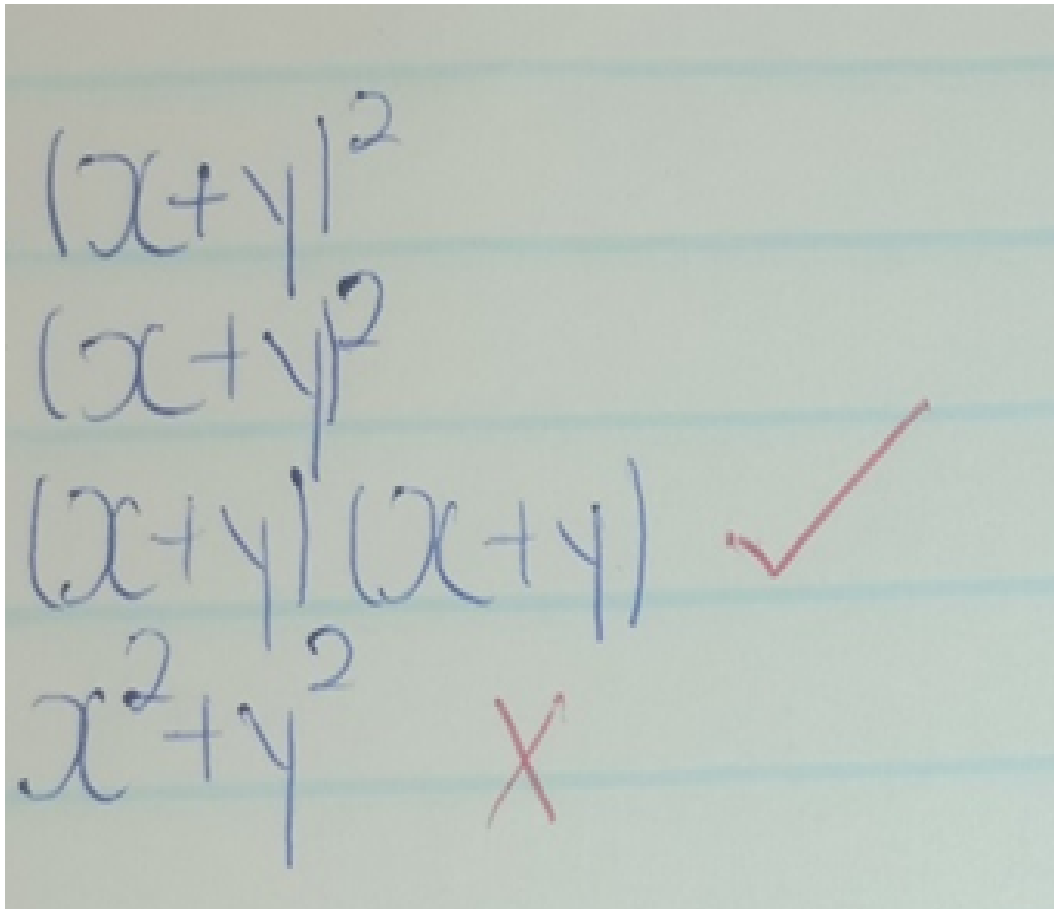


FIGURE 1. Scan 1: A participant's written response of a sampled task

asked, namely, “how” to allow participants to explain their solutions and “why” where necessary during their explanations to monitor and evaluate their reasoning. This guided the researchers to monitor and assess each participants’ knowledge constructions, to identify their misconceptions and to evaluate how identified misconceptions retarded participants’ conceptual understanding of algebraic expressions. Only a handful of extracts from the unstructured interviews are presented in this section to minimise voluminous narrations; each of these are typical of a broader set of narrations. These are presented below.

Scenario 1.

In Scan 1, it can be observed that one response is correct while the other response is wrong. This indicates that the participant partly showed mastery of simplifying simple algebraic expressions- this is evidence which illustrates that this participant operated at the “uni-structure level” of the SOLO model.

$$\frac{ma + mb}{m + md}$$

$$\frac{a + b}{d}$$

FIGURE 2. Scan 2: A participant's written response of a sampled task

Unstructured interview (see Scan 1)

Researcher: Thandi (Not real name), can you explain how you came by your solution?

Thandi: Yes Sir. You see square means multiplies itself twice. So, it becomes. I then expanded the two brackets to get the answer.

Researcher: Thank you Thandi, can you explain why you wrote $x^2 + y^2$ as your final answer.

Thandi: x times x is x^2 and y times y is y^2 .

Researcher: Yes Thandi, it is true that x times x is x^2 and y times y is y^2 . But can you see that all the terms in the two brackets are multiplying and that there is a plus between the x and y in each bracket?

Thandi: Yes sir, that is why I put a plus between the x^2 and y^2 .

The researcher felt that asking Thandi this question will make her realise her mistake for her to correct it by herself but she defended what she wrote.

Scenario 2.

In Scan 2, it can be observed that both the procedure and answer are very wrong, but to the researchers' concern, they have been marked correct by the teacher. It is obvious that this participant had the conception that a cancellation had to be done, although this was not executed appropriately.

Unstructured interview (see Scan 2)

Researcher: Romeo (Not real name), can you explain how you came by your solution.

Romeo: okay. Eeeeeeee m and m are the same so they will cancel each other The second m and m will also cancel. After cancelling, I wrote the terms that are left as the final answer since they cannot be simplified again.

Researcher: Alright Romeo. You put a plus between a and b at the numerator. There is also a plus between m and md at the denominator. Why didn't you bring the plus at the denominator when you wrote your final answer?

Romeo: After cancelling out the common terms, I can see only the d at the denominator.

Romeo saw nothing wrong with his answer after being interrogated by the researcher, but instead, he defended it.

Scenario 3.

In scan 3, it can be seen that the participant rightly factorized the common terms at the numerator and also at the denominator. Thereupon, she lacked the understanding of the next step, thus, could not state the correct answer - an illustration that this participant operated at "uni-structure level" of the SOLO model.

Scenario 4.

Scan 4 informs that the participant correctly factorized the common terms at the numerator and also at the denominator but wrongly applied the mathematical concept "difference of two squares". To the dismay of the researchers, this wrong application was marked correct by the teacher. Scan 4 highlights that there is evidence of one correct response (the first step), which is an illustration that this participant is operating at the "uni-structure level" of the SOLO model.

Scenario 5.

Scan 5, illustrates that the participant knows the correct procedure to follow by trying to factorize the common terms at the numerator and at the denominator, however, this was wrongly executed.

Scenario 6.

$$\begin{aligned}
 & \frac{3x+9}{9+3x^2} \\
 &= \frac{3(x+3)}{3(3+x^2)} \quad \checkmark \\
 &= \frac{x+3}{3+x^2} \\
 &= 3^{1-1} \cdot x^{1-2} \quad \times \\
 &= 3^0 \cdot x^{-1} \\
 &= x^{-1} \\
 &= \frac{1}{x}
 \end{aligned}$$

FIGURE 3. Scan 3: A participant's written response of a sampled task

In scan 6, it can be seen that the correct procedure was applied. The participant could factorize correctly the given expressions at both the numerator and denominator for all the three fractions that constituted the algebraic expression question with the exception of one factorization. That is, the denominator of

$$\begin{aligned}
 & 1.18 \quad \frac{3x+9}{9+3x^2} \\
 & \quad \frac{3(x+3)}{3(3+x^2)} \quad \checkmark \\
 & = \frac{x+3}{3+x^2} = \frac{(x+3)}{(3-x)(3+x)} = \frac{1}{3-x} \quad \checkmark
 \end{aligned}$$

FIGURE 4. Scan 4: A participant's written response of a sampled task

the third fraction should be $2a(1 + 2b)$ instead of $2a(1 + b)$. This is evidence to testify that the participant operated at the uni-structure level of the SOLO model. This is because a mastery of one idea-factorization was demonstrated by the participant.

Scenario 7.

Scan7 depicts that the participant was able to factorize all the terms at the numerators and denominators of the three fractions. Most of the responses were correct, however, the participant could not meaningfully connect ideas from the factorized terms, thus, she could not give an appropriate answer to the given algebraic question. To this end, the researchers rated that this participant was operating at the multi-structure level of the SOLO model. This is because many, but not all, correct responses were given by the participant. Thus, the participant demonstrated mastery of more than one idea – factorization of algebraic expressions and making meaningful connections among the factorized terms. However, this participant omitted $(a - 1)$ at the final answer, resulting to the wrong answer.

3.6. Stage 2.

According to [2] the SOLO model observes, examines and reflects students' learning outcomes. In this study, different degree of questions based on the thinking structures were put together to constitute the standardized test administered to participants and their written responses were collected after the allocated duration. Thereupon, content analysis of their written responses was undertaken holistically. Participants' written responses were then categorised according to

$$\begin{aligned}
 & 1. \quad \frac{x^2 - 1}{x^2 - 2x + 1} \\
 & = \cancel{x^2} (1 + (-1)) \\
 & \quad \cancel{x^2} (-2x + 1) \quad \times \\
 & = \frac{1 + (-1)}{-3} \\
 & = \frac{0}{-3} \quad \times \\
 & = \underline{0} \rightarrow
 \end{aligned}$$

FIGURE 5. Scan 5: A participant's written response of a sampled task

Handwritten mathematical work for Scan 6. The work shows several algebraic expressions and operations, with some parts marked as incorrect with a red 'X'.

$$\begin{array}{l}
 \frac{5a - 5ab}{2b^2 + b - 3} \times \frac{6 + 4b}{5b^2 + 10b + 5} \div \frac{2a + 4ab}{2b^2 + 3b + 1} \\
 \frac{5a(\cancel{1-5})}{(2b+3)(\cancel{b-1})} \times \frac{2(\cancel{3+2b})}{5(b+1)(b+1)} \times \frac{(2b+1)(\cancel{b+1})}{2a(\cancel{1+5})} \\
 \frac{10a(2b+1)}{10a(b+1)^2} \quad \text{X} \\
 \frac{2b+1}{(b+1)^2} \quad \text{X}
 \end{array}$$

FIGURE 6. Scan 6: A participant's written response of a sampled task

Handwritten mathematical work for Scan 7. The work shows several algebraic expressions and operations, with some parts marked as incorrect with a red 'X'.

$$\begin{array}{l}
 \frac{x^2 + 2x + 4}{x^2y^2 - 1} \div \frac{x^3 - 8}{x^2y + x} \times \frac{a^2 + 9}{a^2 - 9} \\
 \frac{x^2 + 2x + 4}{(xy - 1)(xy + 1)} \times \frac{x^2y + x}{x^2 - 8} \times \frac{a^2 + 9}{a^2 - 9} \\
 \frac{x^2 + 2x + 4}{(xy - 1)(xy + 1)} \times \frac{x(xy + 1)}{(x - 2)(x^2 + 2x + 4)} \times \frac{a(a + 1)}{a(a - 1)} \\
 \frac{x(a + 1)}{(xy - 1)(x - 2)} \quad \text{X}
 \end{array}$$

FIGURE 7. Scan 7: A participant's written response of a sampled task

the SOLO levels. In doing so, the following rating scale were applied: pre-structure level = 0, uni-structure level=1, multi-structure level =2, relational level =3. This is presented in Table 2. The extended abstract level was excluded as testing hypothesis and making predictions are not instructional aims and/or objectives of algebraic expressions, as detailed in South Africa's mathematics curriculum[2, 9]. How the SOLO model was applied in this study to analyse participants written and/or oral responses using the four categorization hierarchical abstraction/complexity levels of learning/thinking structures; they are presented in Table 1 [19, 2]. .

Table 2 informed us that: 16 (19%) of participants operated at the pre-structure level; 42 (49%) of participants operated at the uni-structure level; 28 (33%) of participants operated at the multi-structure level and no participant: 0 (0%) of participants operated at the relational level. From the information displayed in

TABLE 1. SOLO model structures - Adapted from[19, 2]

Learning/ Thinking structure levels	Description and expected responses
1. Pre-structure level	No or irrelevant responses. S/he is clueless- does not completely know how to simplify algebraic expressions
2. Uni-structure level	One response: inconsistent responses. Scant evidence of thinking, meaningful constructions and conclusions. S/he shows minimal ability of simplifying algebraic expressions - multiplying a binomial by a trinomial, factorising trinomials, factorising the difference and sums of two cubes, factorising by grouping in pairs, and simplifying, adding and subtracting algebraic fractions with denominators of sum and difference of cubes. Few ideas which are applied are correct.
3. Multi-structure level	S/he generates multiple ideas but ideas are plagued with lack of meaningful connections, medium understanding and thinking - s/he does better at this level than the uni-structural level. S/he show substantial evidence of simplifying algebraic expressions: s/he is able to multiply a binomial by a trinomial, factorize trinomials, factorize differences and sums of two cubes, factorize by grouping in pairs, and to simplify, add and subtract algebraic fractions with denominators of sum and difference of cubes. Most written and/or oral responses are correct.
3. Relational level	Perfect responses; generating relevant ideas and making meaningful connections amongst them; ample evidence of understanding and thinking about algebraic expressions.

Table 2, the researchers focused on the written responses of participants who operated at the uni-structural level - 42 (49%) of participants - as the researchers observed that students were more prone to errors and misconceptions and provided mostly incorrect responses at this crucial level of the SOLO model. This is because participants at the pre-structure level who provided no/irrelevant responses showed evidence of totally lack of ideas and direction, hence, were irrelevant to this study. Participants who operated at the multi-structure and relational levels

TABLE 2. Table 2: Summary of participants' learning outcomes based on the SOLO model levels.

	Pre-structure level	Uni-structure level	Multi-structure level	Relational level
No. of participants	16	42	28	0
Percentages	19%	49%	33%	0%

displayed almost perfect correct responses and excellent applicability of the algebraic expressions and expounded their thoughts explicitly, thus, the researchers considered them as superfluous in realising the aims of this study.

4. FINDINGS AND DISCUSSIONS

The findings that emanated from this study are presented and explicated in line with the formulated research questions.

4.1. Which misconceptions are participants faced with as they learn and solve algebraic expressions problems?

As contained in Table 2, this study established that 16 (19%) of participants operated at the pre-structure level; 42 (49%) operated at the uni-structure level; and 28 (33%) of participants operated at the multi-structure level of the SOLO model. This imply that 58 (67%) of participants either provided no responses, irrelevant responses or minimal evidence of correct responses. This is in accord with literature that students encounter difficulties with algebra [19, 33, 10]. This assertion was established in this study, justifying the rational for conducting this study. The application of the SOLO model in this study guided us to comprehensively understand the sources of errors and misconceptions participants encountered in learning algebraic expressions. This is because the SOLO model assisted us to monitor participants' cognitive development and levels of response. Thus, we were able to assess participants' levels of thinking with reference to the thinking structures of the SOLO model based on their levels of response [20]. According to [10] "the abstract structure of algebraic expressions poses many problems to students such as understanding or manipulating them according to accepted rules, procedures or algorithms". Hence, in this study, the lapses in participants' understanding and thinking with reference to the thinking structures of the SOLO model as they learnt and solved problems in algebraic expressions were identified.

As said earlier, the written responses of 42 (49%) of participants who operated at the uni-structural level of the SOLO model were deeply scrutinised to identify their misconceptions during the unstructured interviews conducted. At this juncture, the nature of the identified msconceptions and the flaws in the misconceptions were presented [21]. The researchers established from this study that

conceptual misunderstandings were predominant among participants' misconceptions in algebraic expressions as most of their written and oral responses from the unstructured interviews were clouded with misinterpretations and misapplications [10]. This is in line with literature as [14] assert that misconceptions are more in relation to students' inability to understand, other than fluency or memory. This author, further, aver that a misconception is a term used to describe deep, intuitive misunderstandings of mathematical concepts. This was based on the evidence that students may know facts, procedures and concepts, however, they may not recollect and apply them at the appropriate moments [15, 25]. According to [25] conceptual misunderstanding is "a type of misconception that arise when students are taught scientific information in a way that does not provoke them to confront paradoxes and conflicts resulting from their own preconceived notions and non-scientific beliefs".

As justification for this research finding, the researchers make reference to scan 1, as it can be observed that one response is correct while the other response is wrong. This testifies that the participant was operating at the "one-structure level" of the SOLO model. This participant correctly interpreted $(x + y)^2 = (x + y)(x + y)$. However, he ended up, with the wrong answer as $x^2 + y^2$. He could not expand and simplify the terms in the two brackets. This is evidence of misinterpretations and misapplications of algebraic concepts, which led to this conceptual misunderstanding. Also, on Scan 2, it can be observed that both the procedure and answer are very wrong. The researchers aver that conceptual misunderstanding resulted in the participant applying the inappropriate procedure. [29] avers that "misconceptions are indicators of poor understanding". In addition, [22] posit that conceptual misunderstanding leads to misconceptions. [16] add that misconceptions can be conceptual, procedural or application-oriented.

On scan 3, the participant was able to factorize the common terms at the numerator and also at the denominator, but could not realise at this point, that the terms cannot be simplified further, hence, they should have been left as the final answer; this is another evidence of conceptual misunderstanding. It was evident that this participant operated at the "one-structure level" of the SOLO model as one idea was demonstrated. On scan 4, the participant correctly factorized the common terms at the numerator and also at the denominator but wrongly applied the mathematical concept "difference of two squares". This is another evidence to testify that participants' work was dominated by conceptual misunderstandings - misinterpretations and misapplications. On Scan 5, the participant knew the procedure - she attempted to factorize the common terms at the numerator and at the denominator, however, conceptual misunderstandings, resulted in this participant not being able to apply the procedure correctly. In addition, on scan 6, the participant correctly cancelled out $2b + 3$ (at the denominator of the first fraction) and $3 + 2b$ (numerator of the second fraction). From the third fraction he also correctly cancelled $1 + 2b$ and $2b + 1$, however, he went ahead to wrongly cancel out $1 - b$ and $b - 1$ (from the first fraction). It can be seen on scan 6 that

the teacher put wrong marks on both $1 - b$ and $b - 1$; indicating that the cancellations were incorrect. This is another evidence of conceptual misunderstandings which resulted from misinterpretations and misapplications of algebraic concepts. This was displayed by a participant who operated at the uni-structure level of the SOLO model as one idea was demonstrated. In addition, on scan 7, it is evident that all the factorized terms were correct, although $(a - 1)$ was left out at the denominator when writing the correct answer. This is another evidence to justify that conceptual misunderstanding was dominant in participants' work. This has resulted in the participant not obtaining the appropriate answer. This gave the researchers a reason to categorize this participant as "operating at the multi-structure level of the SOLO model".

Furthermore, on scan 2, it is evident that both the procedure and answer are wrong, but to the researchers' worry, they were marked correct by the teacher. Also, on scan 4 it was detected that the teacher marked a wrong misinterpretation and misapplications of the mathematical concept "difference of two squares" as correct; also in scan 6, a wrong factorization $2a(1 + b)$ was marked right by the teacher. These evidences might be construed that the teacher himself has his own misconceptions. [24, 18] maintains that misconceptions that teachers harbour are extendable to their students. This implies that not only students are prone to misconceptions, but also, teachers, thus, effective instructional approaches need to be implemented at teacher training institutions to break the circle of teachers transferring their misconceptions to their students [1, 22, 27]

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4.2. How do the identified misconceptions hinder participants conceptual understanding of algebraic expressions?

Conceptual misunderstandings comprising of misinterpretations and misapplications were participants' main misconceptions in algebraic expressions identified in this study. [33] described misconceptions as "one of the most important barriers in learning mathematics". We ascertained from this study that these identified misconceptions impeded participants' conceptual understanding of algebraic concepts. Thus, how conceptual misunderstandings hinder participants' conceptual understanding are delineated in two facets. Firstly, the researchers established that participants were unable to develop conceptualization and appropriate meanings of algebraic expressions as they could not form appropriate ideas in their minds, however, developing conceptualization and appropriate meanings is integral to developing conceptual understanding [32, 17]. For instance, on scan 1, a participant had the knowledge that $(x + y)(x + y) = x^2 + y^2$; on scan 2, it can be observed that the participant had challenges simplifying a simple algebraic expression as both the procedure and answer were not appropriate; in scan 4 the participant also wrongly interpreted and applied the mathematical concept "difference of two squares". These misconceptions impeded participants' conceptualization of algebraic expressions as they formed inappropriate meanings of those concepts. These participants who harboured these misconceptions were

incapacitated in solving non-routine algebra problems (see scans 6 and 7) and algebraic proofs which are heavily reliant on students' conceptions of these integral algebraic concepts.

Secondly, conceptual misunderstandings did not invoke students' rational reasoning. According to [23] "As students move from arithmetic to algebra, they are entering a new world of abstract thought, unknown quantities and latent relationships". Also, according to [26] "misconceptions are misunderstandings and misinterpretations based on incorrect meanings impeding students' rational reasoning". This served as literature backing to this research finding. In justifying this research finding, we make reference to scan 5; the participant had the notion of factorizing the common terms at the numerator and at the denominator but this was inappropriately carried out. Also, the participant stating that $-2x + 1 = -3$ is worrisome (see scan 5). As these participants have internalized these inappropriate ideas of relevant algebraic concepts, they were not effectively able to brainstorm around relevant algebraic concepts to solve non-routine algebraic expression problems (see scans 6 and 7).

This is because these participants were unable to link their thoughts in the abstract domain, which is counter-intuitive [23]. The aforesaid extract from scan 5 also demonstrates that this participant's written responses were devoid of analysis due to inability to make appropriate connections between ideas and mathematical structures (also see scans 2 and 3). With synthesis, participants were unable to apply mathematical ideas in different contexts (see scan 4), in addition, the participant was unable to apply the mathematical concept of "difference of two squares" in the given context. For evaluation, participants' written responses as displayed on scans 1-5 were evidence that they were inept to know how concepts and procedures are interrelated and associated and to meaningfully justify them. We assert that if participants had the ability to justify ideas, some of their misconceptions could have been corrected by themselves. For instance, if these participants had expanded and simplified the terms in these two brackets $(x + y)(x + y)$ they would have known that this will not be equal to $x^2 + y^2$. Analysis, synthesis and evaluation, according to [15] are integral to developing students' conceptual understanding which participants lacked, hence, participants were not able to make sense of these algebraic concepts, which impeded their algebraic reasoning [17]. [28] maintains that such these answers clearly inform that participants do not have "relational understanding". Participants' work demonstrated that they learnt algebraic expressions by learning rules and procedures; this is what [28] called "instrumental understanding". Participants had tried to generalise these rules and procedures to solve every problem that came their way but this did not work out owing to different contexts.

5. CONCLUSIONS AND RECOMMENDATION

Mathematical concepts remain unchanged over the years. However, the way they are taught and/or learnt makes students learn unexpected mathematical concepts

which deviate from the mathematical truth. This becomes encumbrance to attaining conceptual understanding. This study has established that students are faced with misconceptions in algebra and the main misconception identified in this study is conceptual misunderstanding. This conclusion is justified in this study as participants were unable to recollect and apply algebraic concepts at the appropriate moments. The researchers further established that these identified misconceptions impeded students' conceptual understanding in the following ways: (1) students are unable to develop conceptualization and appropriate meanings of an algebraic concept as they are unable to form appropriate ideas in their minds and (2) conceptual misunderstandings do not elicit students' rational reasoning. This study concluded that the root cause of these misconceptions can be attributed to ineffective pedagogical approaches. The researchers, thus, recommend that effective instructional approaches must be lectured on during training of teachers so that they can subsequently implement them in Mathematics classrooms to guide students from developing these misconceptions. Therefore, in conclusion, the researchers issued a caveat that unless effective instructional approaches are implemented in mathematics classrooms, students' misconceptions and incessant under-performance in algebraic expressions cannot be ameliorated.

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