

APPROXIMATE FIXED POINT THEOREMS IN GENERALIZED METRIC SPACES

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ABSTRACT. In this paper, we prove approximate fixed point theorems for various types of contraction mappings in the framework of generalized metric spaces by introducing the concept of T -asymptotically regular mapping. Our results in this paper extends and improves upon, among others, the corresponding results of Berinde and Prasad et al.

1. INTRODUCTION AND PRELIMINARIES

Fixed point theorems have been proved to be useful instrument in many applied areas in mathematics. However, for many practical situations the conditions that have to be imposed in order to guarantee the existence of fixed points are too strong. In such situations an approximate fixed point is more than enough, and approximate solution plays an important role for the requirement of the practical purposes. If X is a metric space, approximate fixed point theorems are interesting. Suppose that we would like to find an approximate solution of $Tx = x$. If there exists a point $x_0 \in X$ such that $d(Tx_0, x_0) < \epsilon$, where ϵ is a positive number, then x_0 is called an approximate solution of the equation $Tx = x$ or we can say that $x_0 \in X$ is an approximate fixed point (or ϵ -fixed point) of T .

Considerable amount of research have been done on approximate fixed point property for various types of mappings for the last few years. In 2006, Berinde [2] proved quantitative and qualitative approximate fixed point theorems for various types of well known contractions on metric spaces. It was proved that even by weakening the conditions by giving up the completeness of the space, the existence of ϵ -fixed points is still guaranteed for operators satisfying Kannan, Chatterjea and Zamfirescu type of conditions on metric spaces. In 2009 Prasad et al. [11] extended those results to generalized metric space.

In 2009, Beiranvand, et al. [1] introduced the notions of T -Banach contraction and T -contractive mapping and extended the Banach contraction principle [3] and Edelstein's fixed point theorem [5]. In the same year, Moradi [7] introduced

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the T -Kannan contractive type mappings, extending in this way the well-known Kannan's fixed point theorem given in [6]. The corresponding versions of T -contractive, T -Kannan mappings and T -Chatterjea contractions on cone metric spaces was studied in [10]. The same authors [9], then studied the existence of fixed points of T -Zamfirescu and T -weak contraction mappings defined on a complete cone metric space. Later, in [8] they studied the existence of fixed points for T -Zamfirescu operators in complete metric spaces and proved a convergence theorem of T -Picard iteration for the class of T -Zamfirescu operators.

Inspired and motivated by the above facts, we prove approximate fixed point theorems for the classes of T -Banach contraction, T -Kannan contraction, T -Chatterjea contraction, T -Zamfirescu operators and in b-metric spaces. Here we introduce the concept of T -asymptotically regular mapping in a b-metric space and then establish a lemma regarding approximate fixed points of the commuting mappings in b-metric spaces. We use this lemma to prove qualitative theorems for various types of contractions in b-metric spaces.

We need the following definitions to prove our main results:

Definition 1.1. Let X be a non empty set and $s \geq 1$ be a given real number. A function $d : X \times X \rightarrow R_+$ (set of non negative real numbers) is said to be a b-metric iff for all $x, y, z \in X$ the following conditions are satisfied:

- (1) $d(x, y) = 0$ iff $x = y$
- (2) $d(x, y) = d(y, x)$
- (3) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

The pair (X, d) is called a b-metric space.

The class of b-metric spaces is effectively larger than that of metric spaces, since a b-metric is a metric space when $s = 1$ in above condition (iii). But a b-metric on X need not be a metric on X [11].

Definition 1.2. Let (X, d) be a metric space. Let $f : X \rightarrow X, \epsilon > 0$ and $x \in X$. Then x_0 is an ϵ -fixed point (approximate fixed point) of f if

$$d(f(x_0), x_0) < \epsilon.$$

Note. The set of all ϵ -fixed points of f , for a given ϵ can be denoted by $F_\epsilon(f) = \{x \in X : x \text{ is an } \epsilon - \text{fixed point of } f\}$.

Definition 1.3. Let (X, d) be a metric space and $f : X \rightarrow X$. Then f has the approximate fixed point property (a.f.p.p) if for every $\epsilon > 0$,

$$F_\epsilon(f) \neq \phi.$$

Definition 1.4. Let (X, d) be a metric space, $f : X \rightarrow X$ is said to be asymptotically regular if

$$d(f^n(x), f^{n+1}(x)) \rightarrow 0 \text{ as } n \rightarrow \infty, \quad \forall x \in X.$$

Definition 1.5. Let (X, d) be a metric space, $T, S : X \rightarrow X$ be two functions. S is called T -asymptotically regular if

$$d(TS^n(x), TS^{n+1}(x)) \rightarrow 0 \text{ as } n \rightarrow \infty, \quad \forall x \in X.$$

Definition 1.6. Let (X, d) be a metric space and $T, S : X \rightarrow X$ be two functions. S is said to be T -**Banach contraction** (TB contraction) if there exists $a \in [0, 1)$ such that

$$d(TSx, TSy) \leq ad(Tx, Ty), \quad \forall x, y \in X.$$

If we take $T = I$, the identity map, then we obtain the definition of *Banach's contraction* [3].

Definition 1.7. Let (X, d) be a metric space and $T, S : X \rightarrow X$ be two functions. S is said to be T -**Kannan contraction** (TK contraction) if there exists $b \in [0, \frac{1}{2})$ such that

$$d(TSx, TSy) \leq b[d(Tx, TSx) + d(Ty, TSy)], \quad \forall x, y \in X.$$

Here when $T = I$, the identity map, we get *Kannan operator* [6].

Definition 1.8. Let (X, d) be a metric space and $T, S : X \rightarrow X$ be two functions. S is said to be T -**Chatterjea contraction** (TC contraction) if there exists $c \in [0, \frac{1}{2})$ such that

$$d(TSx, TSy) \leq c[d(Tx, TSy) + d(Ty, TSx)], \quad \forall x, y \in X.$$

When $T = I$, the identity map, in the above definition, it becomes *Chatterjea Operator* [4].

Definition 1.9. Let (X, d) be a metric space and $T, S : X \rightarrow X$ be two functions. S is said to be T -**Zamfirescu operator** (TZ operator) if there are real numbers $0 \leq a < 1, 0 \leq b < \frac{1}{2}, 0 \leq c < \frac{1}{2}$ such that for all $x, y \in X$ at least one of the conditions is true:

$$(TZ_1) : d(TSx, TSy) \leq ad(Tx, Ty), (TZ_2) : d(TSx, TSy) \leq b[d(Tx, TSx) + d(Ty, TSy)], (TZ_3) : d(TSx, TSy) \leq c[d(Tx, TSy) + d(Ty, TSx)],$$

When the function T is equated to I , the identity map, we obtain the definition of Zamfirescu operator introduced in [13].

In order to prove our main results we need the following lemma:

Lemma 1.10. Let (X, d) be a b -metric space and $T, S : X \rightarrow X$ be two commuting mappings. If S is T -asymptotically regular, then S has approximate fixed point property.

Proof. Let $x_0 \in X$. Since $S : X \rightarrow X$ is T -asymptotically regular, we have

$$d(TS^n(x_0), TS^{n+1}(x_0)) \rightarrow 0 \text{ as } n \rightarrow \infty, \quad \forall x \in X,$$

which gives that for every $\epsilon > 0$, there exists $n_0(\epsilon) \in \mathbb{N}$ such that

$$d(TS^n(x_0), TS^{n+1}(x_0)) < \epsilon, \quad \forall n \geq n_0(\epsilon)$$

i.e.,

$$d(TS^n(x_0), TS(S^n(x_0))) < \epsilon, \quad \forall n \geq n_0(\epsilon).$$

Since T and S are commuting mappings, this implies that for every $\epsilon > 0$, there exists $n_0(\epsilon) \in \mathbb{N}$ such that

$$d(TS^n(x_0), ST(S^n(x_0))) < \epsilon, \quad \forall n \geq n_0(\epsilon).$$

Denote $y_0 = TS^n(x_0)$. Then we get that for every $\epsilon > 0$, there exists $y_0 \in X$ such that

$$d(y_0, Sy_0) < \epsilon, \quad \forall n \geq n_0(\epsilon).$$

So for each $\epsilon > 0$, there exists an ϵ -fixed point of S in X , namely y_0 which means that S has approximate fixed point property. $\square$

2. MAIN RESULTS

Theorem 2.1. *Let (X, d) be a b -metric space and $T, S : X \rightarrow X$ be two commuting mappings where S is a TB -contraction. Then for every $\epsilon > 0$,*

$$F_\epsilon(S) \neq \emptyset,$$

i.e., S has approximate fixed point property.

Proof. Let $\epsilon > 0, x \in X$. Then

$$\begin{aligned} d(TS^n(x), TS^{n+1}(x)) &= d(TS(S^{n-1}(x)), TS(S^n(x))) \\ &\leq ad(TS^{n-1}(x), TS^n(x)) \\ &\leq a^2d(TS^{n-2}(x), TS^{n-1}(x)) \\ &\leq \dots \leq a^nd(Tx, TSx) \end{aligned}$$

Since $a \in [0, 1)$, from the above inequality we get that

$$d(TS^n(x), TS^{n+1}(x)) \rightarrow 0 \text{ as } n \rightarrow \infty, \quad \forall x \in X,$$

which implies that S is T -asymptotically regular. Now by applying Lemma 1.1 we obtain that for every $\epsilon > 0$,

$$F_\epsilon(S) \neq \emptyset,$$

which means that S has approximate fixed point property. $\square$

If $s=1$, we get the following result obtained in [12]:

Corollary 2.2. [12, Theorem 2.1]

Let (X, d) be a metric space and $T, S : X \rightarrow X$ be two commuting mappings where S is a TB -contraction. Then for every $\epsilon > 0$,

$$F_\epsilon(S) \neq \emptyset,$$

i.e., S has approximate fixed point property.

If we put $T=I$, we obtain the following result of Prasad et al. [11]:

Corollary 2.3. [1, Theorem 3.2] *Let (X, d) be a b -metric space and $T : X \rightarrow X$ an a -contraction. Then for every $\epsilon > 0$, the diameter of $F_\epsilon(T)$ is not larger than $\frac{s\epsilon(1+s)}{1-as^2}$.*

When $T=I$, $s=1$ we obtain the following results of Berinde [2]:

Corollary 2.4. [2, Theorem 2.1] *Let (X, d) be a metric space and $f : X \rightarrow X$ an a -contraction. Then for every $\epsilon > 0$,*

$$F_\epsilon(f) \neq \emptyset.$$

Corollary 2.5. [2, Theorem 3.1] *Let (X, d) be a metric space and $f : X \rightarrow X$ an a -contraction. Then for every $\epsilon > 0$, the diameter of $F_\epsilon(f)$ is not larger than $\frac{2\epsilon}{1-a}$.*

Theorem 2.6. *Let (X, d) be a b -metric space and $T, S : X \rightarrow X$ be two commuting mappings where S is a TK -contraction. Then for every $\epsilon > 0$,*

$$F_\epsilon(S) \neq \emptyset,$$

i.e., S has approximate fixed point property.

Proof. Let $\epsilon > 0, x \in X$. Then

$$\begin{aligned} d(TS^n(x), TS^{n+1}(x)) &= d(TS(S^{n-1}(x)), TS(S^n(x))) \\ &\leq b[d(TS^{n-1}(x), TS(S^{n-1}(x))) + d(TS^n(x), TS(S^n(x)))] \\ &= b[d(TS^{n-1}(x), TS^n(x)) + d(TS^n(x), TS^{n+1}(x))]. \end{aligned}$$

Thus we have the inequality

$$(1-b)d(TS^n(x), TS^{n+1}(x)) \leq b[d(TS^{n-1}(x), TS^n(x))],$$

which implies that

$$\begin{aligned} d(TS^n(x), TS^{n+1}(x)) &\leq \frac{b}{1-b}d(TS^{n-1}(x), TS^n(x)) \\ &\leq \left(\frac{b}{1-b}\right)^2 d(TS^{n-2}(x), TS^{n-1}(x)) \\ &\leq \dots \leq \left(\frac{b}{1-b}\right)^n d(Tx, TSx). \end{aligned}$$

Since $b \in [0, \frac{1}{2})$, from the above inequality we get that

$$d(TS^n(x), TS^{n+1}(x)) \rightarrow 0 \text{ as } n \rightarrow \infty, \quad \forall x \in X,$$

which implies that S is T -asymptotically regular. Now by applying Lemma 1.1 we obtain that for every $\epsilon > 0$,

$$F_\epsilon(S) \neq \emptyset,$$

which means that S has approximate fixed point property. □

If $s=1$, we get the following result obtained in [12]:

Corollary 2.7. [12, Theorem 2.3] *Let (X, d) be a metric space and $T, S : X \rightarrow X$ be two commuting mappings where S is a TK -contraction. Then for every $\epsilon > 0$,*

$$F_\epsilon(S) \neq \phi,$$

i.e., S has approximate fixed point property.

If we put $T=I$, in above Theorem we obtain the result of Prasad et al. [11].

Corollary 2.8. [11, Theorem 3.4] *Let (X, d) be a b -metric space and $T : X \rightarrow X$ a Kannan mapping. Then for every $\epsilon > 0$, the diameter of $F_\epsilon(T)$ is not larger than $s\epsilon(1 + s + 2bs)$.*

When $T=I$, $s=1$ we obtain the following results of Berinde [2]:

Corollary 2.9. [2, Theorem 2.2] *Let (X, d) be a metric space and $f : X \rightarrow X$ a Kannan mapping. Then for every $\epsilon > 0$,*

$$F_\epsilon(f) \neq \phi.$$

Corollary 2.10. [2, Theorem 3.2] *Let (X, d) be a metric space and $f : X \rightarrow X$ a Kannan mapping. Then for every $\epsilon > 0$, the diameter of $F_\epsilon(T)$ is not larger than $2\epsilon(1 + b)$.*

Theorem 2.11. *Let (X, d) be a b -metric space and $T, S : X \rightarrow X$ be two commuting mappings where S is a TC -contraction. Then for every $cs < 1/2$ and $\epsilon > 0$,*

$$F_\epsilon(S) \neq \phi,$$

i.e., S has approximate fixed point property.

Proof. Let $\epsilon > 0, x \in X$. Then

$$\begin{aligned} d(TS^n(x), TS^{n+1}(x)) &= d(TS(S^{n-1}(x)), TS(S^n(x))) \\ &\leq c[d(TS^{n-1}(x), TS(S^n(x))) + d(TS^n(x), TS(S^{n-1}(x)))] \\ &= c[d(TS^{n-1}(x), TS^{n+1}(x)) + d(TS^n(x), TS^n(x))] \\ &= c[d(TS^{n-1}(x), TS^{n+1}(x))]. \end{aligned}$$

$$d(TS^n(x), TS^{n+1}(x)) \leq cs[d(TS^{n-1}(x), TS^n(x)) + d(TS^n(x), TS^{n+1}(x))],$$

which gives

$$(1 - cs)d(TS^n(x), TS^{n+1}(x)) \leq cs[d(TS^{n-1}(x), TS^n(x))].$$

Thus we have the inequality,

$$\begin{aligned} d(TS^n(x), TS^{n+1}(x)) &\leq \frac{cs}{1-cs} d(TS^{n-1}(x), TS^n(x)) \\ &\leq \left(\frac{cs}{1-cs} \right)^2 d(TS^{n-2}(x), TS^{n-1}(x)) \\ &\leq \dots \leq \left(\frac{cs}{1-cs} \right)^n d(Tx, TSx). \end{aligned}$$

This implies

$$d(TS^n(x), TS^{n+1}(x)) \rightarrow 0 \text{ as } n \rightarrow \infty, \quad \forall x \in X,$$

which implies that S is T -asymptotically regular. Now by applying Lemma 1.1 we obtain that for every $\epsilon > 0$,

$$F_\epsilon(S) \neq \phi,$$

which means that S has approximate fixed point property. $\square$

If $s=1$, we get the following result obtained in [12]:

Corollary 2.12. [12, Theorem 2.5] *Let (X, d) be a metric space and $T, S : X \rightarrow X$ be two commuting mappings where S is a TC-contraction. Then for every $\epsilon > 0$,*

$$F_\epsilon(S) \neq \phi,$$

i.e., S has approximate fixed point property.

If we put $T=I$, in Theorem 2.9 we obtain the following result obtained by Prasad et al . [11]:

Corollary 2.13. [11, Theorem 3.6] *Let (X, d) be a b -metric space and $T : X \rightarrow X$ a Chatterjea operator. Then for every $\epsilon > 0$, the diameter of $Fix_\epsilon(T)$ is not larger than $\frac{s\epsilon(1+s+2cs)}{1-2s^2c}$.*

When $T=I$, $s=1$ we obtain the following results of Berinde [2]:

Corollary 2.14. [2, Theorem 3.3] *Let (X, d) be a b -metric space and $T : X \rightarrow X$ a Chatterjea operator. Then for every $\epsilon > 0$, the diameter of $Fix_\epsilon(T)$ is not larger than $\frac{2\epsilon(1+c)}{1-2c}$.*

Corollary 2.15. [2, Theorem 2.3] *Let (X, d) be a metric space and $f : X \rightarrow X$ a Chatterjea operator. Then for every $\epsilon > 0$,*

$$F_\epsilon(f) \neq \phi.$$

Theorem 2.16. *Let (X, d) be a b -metric space and $T, S : X \rightarrow X$ be two commuting mappings where S is a TZ -operator. Then for $bs^2 < 1/2$, $cs < 1/2$, S has approximate fixed point property.*

Proof. Let $\epsilon > 0, x \in X$.

If (TZ_2) holds,

$$\begin{aligned} d(TSx, TSy) &\leq b[d(Tx, TSx) + d(Ty, TSy)] \\ &\leq b[d(Tx, TSx)] + bs[d(Ty, Tx) + d(Tx, TSy)] \\ &= b[d(Tx, TSx)] + bs[d(Ty, Tx)] + bs^2[d(Tx, TSx) + d(TSx, TSy)] \\ &= b(1 + s^2)[d(Tx, TSx)] + bs[d(Tx, Ty)] + bs^2[d(TSx, TSy)]. \end{aligned}$$

The above inequality gives

$$(1 - bs^2)d(TSx, TSy) \leq b(1 + s^2)[d(Tx, TSx)] + bs[d(Tx, Ty)]$$

, which implies

$$d(TSx, TSy) \leq \frac{b(1 + s^2)}{1 - bs^2}[d(Tx, TSx)] + \frac{bs}{1 - bs^2}[d(Tx, Ty)]. \quad (2.1)$$

If (TZ_3) holds, then

$$\begin{aligned} d(TSx, TSy) &\leq c[d(Tx, TSy) + d(Ty, TSx)] \\ &\leq cs[d(Tx, Ty) + d(Ty, TSy)] + cs[d(Ty, TSy) + d(TSy, TSx)] \\ &= cs[d(Tx, Ty)] + 2cs[d(Ty, TSy)] + cs[d(TSx, TSy)]. \end{aligned}$$

The above inequality gives

$$(1 - cs)d(TSx, TSy) \leq 2cs[d(Ty, TSy)] + cs[d(Tx, Ty)],$$

which implies

$$d(TSx, TSy) \leq \frac{2cs}{1 - cs}[d(Ty, TSy)] + \frac{cs}{1 - cs}[d(Tx, Ty)]. \quad (2.2)$$

If TZ_3 holds, we can also have

$$\begin{aligned} d(TSx, TSy) &\leq c[d(Tx, TSy) + d(Ty, TSx)] \\ &\leq cs[d(Tx, TSx) + d(TSx, TSy)] + cs[d(Ty, Tx) + d(Tx, TSx)] \\ &= cs[d(Tx, Ty)] + 2cs[d(Tx, TSx)] + cs[d(TSx, TSy)]. \end{aligned}$$

The above inequality gives

$$(1 - cs)d(TSx, TSy) \leq 2cs[d(Tx, TSx)] + cs[d(Tx, Ty)],$$

which implies

$$d(TSx, TSy) \leq \frac{2cs}{1 - cs}[d(Tx, TSx)] + \frac{cs}{1 - cs}[d(Tx, Ty)]. \quad (2.3)$$

From (TZ_1) , (2.1), (2.2) and (2.3) we can denote

$$\delta = \max \left\{ a, \frac{bs}{1 - bs^2}, \frac{b(1 + s^2)}{2(1 - bs^2)}, \frac{cs}{1 - cs} \right\}.$$

Then we have $0 \leq \delta < 1$ and in view of (TZ_1) , (2.1) and (2.2), it results that the inequalities

$$d(TSx, TSy) \leq 2\delta[d(Tx, TSx)] + \delta[d(Tx, Ty)] \quad (2.4)$$

$$d(TSx, TSy) \leq 2\delta[d(Ty, TSy)] + \delta[d(Tx, Ty)] \quad (2.5)$$

holds for all $x, y \in X$.

Using (2.4), we get

$$\begin{aligned} d(TS^n(x), TS^{n+1}(x)) &= d(TS(S^{n-1}(x)), TS(S^n(x))) \\ &\leq 2\delta[d(TS^{n-1}(x), TS(S^{n-1}(x)))] + \delta[d(TS^{n-1}(x), TS^n(x))] \\ &= 3\delta[d(TS^{n-1}(x), TS^n(x))]. \end{aligned}$$

Thus we obtain that

$$\begin{aligned} d(TS^n(x), TS^{n+1}(x)) &\leq 3\delta[d(TS^{n-1}(x), TS^n(x))] \\ &\leq (3\delta)^2 [d(TS^{n-2}(x), TS^{n-1}(x))] \\ &\leq \dots \leq (3\delta)^n [d(Tx, TSx)]. \end{aligned}$$

Since $\delta \in [0, 1)$, the above inequality gives

$$d(TS^n(x), TS^{n+1}(x)) \rightarrow 0 \text{ as } n \rightarrow \infty, \quad \forall x \in X,$$

which implies that S is T -asymptotically regular. Now by applying Lemma 1.1 we obtain that for every $\epsilon > 0$,

$$F_\epsilon(S) \neq \emptyset,$$

which means that S has approximate fixed point property. $\square$

If $s=1$, we get the following result obtained in [12]:

Corollary 2.17. [12, Theorem 2.7]

Let (X, d) be a metric space and $T, S : X \rightarrow X$ be two commuting mappings where S is a TC -contraction. Then for every $\epsilon > 0$,

$$F_\epsilon(S) \neq \emptyset,$$

i.e., S has approximate fixed point property.

If we put $T=I$, in Theorem we obtain the result of Prasad et al. [11]:

Corollary 2.18. [11, Theorem 3.8] Let (X, d) be a b -metric space and

$T : X \rightarrow X$ a Zamfirescu operator. Then for every $\epsilon > 0$, the diameter of $Fix_\epsilon(S)$ is not larger than $\frac{s\epsilon(1+s+2\delta s)}{1-\delta}$ where $\delta = \max \left\{ a, \frac{bs}{1-bs^2}, \frac{b(1+s^2)}{2(1-bs^2)}, \frac{cs}{1-cs} \right\}$.

When $T=I$, $s=1$ we obtain the following results of Berinde [2]:

Corollary 2.19. [2, Theorem 3.4] *Let (X, d) be a b -metric space and $T : X \rightarrow X$ a Zamfirescu operator. Then for every $\epsilon > 0$, the diameter of $\text{Fix}_\epsilon(T)$ is not larger than $\frac{2\epsilon(1+2\delta)}{1-\delta}$ where $\delta = \max \left\{ a, \frac{b}{1-b}, \frac{c}{1-c} \right\}$.*

Corollary 2.20. [2, Theorem 2.4] *Let (X, d) be a metric space and $f : X \rightarrow X$ a Zamfirescu operator. Then for every $\epsilon > 0$,*

$$F_\epsilon(f) \neq \phi.$$

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