USING EXPERIENTIAL LEARNING IN SOLVING 3-DIMENSIONAL PROBLEMS IN TRIGONOMETRY

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ABSTRACT. In this case study, the efficacy of experiential learning for solving 3-dimensional problems in trigonometry was tested. The study involved two groups of students in grade 12 at a high school in South Africa, the control (n=35) and the experimental (n= 37). The control group was taught via the traditional approach, whilst the experimental group was taught using experiential learning in a collaborative classroom setting. Kolb’s experiential learning theory was applied as the theoretical framework and data was collected through pre- and post-tests. The pre-test and post-test scores for both groups were analysed using the mixed-effects negative binomial model and from the results it was established that experiential learning resulted in significant improvements in participants’ performance compared to the traditional method. Therefore, the researchers concluded that experiential learning is an effective instructional approach, thus, recommending this mode of pedagogy for solving 3-dimensional problems in trigonometry.

1. INTRODUCTION AND PRELIMINARIES

Roger Bacon, as quoted in [29] once stated: "Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of the world." This is a clarion call advocating for learners to study mathematics owing to its relevance in academia and in our daily lives. Experiential learning might help students to understand, recognise and appreciate the relevance of mathematics in our daily life, as this mode of pedagogy accords students the leverage to interact with their observable experiences. Experiential learning makes mathematics lessons practical, which can dissuade students from developing phobia for mathematics, which could negatively affect their involvement and performance, not only in mathematics, but in other related subjects and careers in science and technology [1, 25]. This is because mathematics serves as the key to the doors of other related subjects since it is part of the fundamental knowledge required for learning other science and commerce subjects [7].

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The relevance of applications of mathematics in our daily life, in academia, and at workplaces has placed much focus on mathematical modelling and creativity \[24, 29\]. However, the appropriate instructional methods for teaching mathematics that would nurture students’ knowledge and skills in mathematical modelling and creativity in the context of solving 3-dimensional problems in trigonometry are lacking in literature. This leaves mathematics teachers to decipher which approach to implement for teaching difficult mathematical contents such as trigonometry \[18\]. To help fill this lacuna in literature, we investigated experiential learning, for solving problems in one of the integral mathematical concepts inherently embedded with mathematical modelling and creativity. Therefore, experiential learning as an instructional approach was investigated in this study to solve 3-dimensional problems in trigonometry. The researchers from their personal experiences as high school mathematics educators consider 3-dimensional problems in trigonometry to be a practical mathematical concept requiring advanced mathematical thinking and applications of relevant trigonometry concepts such as the sine and cosine rules \[26\].

High school students experience severe learning and problem-solving difficulties in relation to trigonometry, especially its associated application-orientated concepts such as 3-dimensional problems \[18\]. The challenges of students in trigonometry are ubiquitous in mathematics classrooms in South Africa, leading to a cacophony of voices calling for immediate intervention \[10\]. Thus, in this study, the intersubjectivity of the researchers led to an investigation of how experiential learning, a contemporary pedagogical approach embedded with mathematical modelling and creativity, can be used in solving 3-dimensional problems in trigonometry \[9\]. This study sought to find an alternative to the traditional approach, as researchers \[18, 26\] urge mathematics teachers to discard rote learning in favour of teaching learners to attain conceptual understanding of trigonometry which would aid them to effectively build their knowledge for them to be able to solve related real-life problems. Thus, some researchers urge mathematics students to endlessly learn, unlearn and relearn required mathematical content knowledge \[4\]. This will make their journey of migrating from the known (prior knowledge) to the unknown (new learning) efficient.

Researchers \[21\] posit that students encounter challenges in applying their knowledge of trigonometry in solving related application problems such as 3-dimensional and real-life problems. This is partly alluded to the notion that students are highly unable to interpret and understand the given information in each trigonometry application question \[21\]. In terms of the APOS theory, researchers \[18\] observed that several learners were at the action level of mental conception when solving 3-dimensional problems. This is because they could only recall basic trigonometry knowledge without demonstrating any substantial evidence of application of trigonometry knowledge in solving 3-dimensional problems. Furthermore, according to \[27\] students encounter transformation error and process skill error in solving problems in trigonometry, in the lens of the Newman’s error analysis model. This further highlight students’ deficits in solving application
questions associated with trigonometry.

Germane to this study is the assertion by [3] that ‘Mathematics teachers should make the teaching of trigonometry practical and more activity-orientated’. These authors emphasised that this would help students overcome their challenges in solving trigonometry application questions. This is the main rationale of this study as we investigated experiential learning for solving problems in 3-dimensions in trigonometry. Experiential learning is defined as ‘sequence of events that include one or more specific study objects that require active involvement of learners in various stages of the process’ [8]. One of the early advocates of experiential learning, John Dewey, in 1916, asserts that give students something to do, not something to learn; and doing is of such nature as to require thinking; learning naturally results’.

In this study, Kolb’s experiential learning theory was applied, incorporating a four-stage learning cycle transformed into effective learning, that is, concrete experience (doing), reflective observation (observing), abstract conceptualisation (thinking), and active experimentation (planning). Experiential learning is an instructional approach which is connoted as learning by” doing” and has been established to be an effective teaching method [15]. This mode of pedagogy guides students to construct their own first-hand knowledge using instructional materials, rather than learning someone’s constructed knowledge and taking them as definite [15]. Also, experiential learning promotes active and reflective learning. Most importantly, it serves as a medium which connects theory (mathematical knowledge) and practice (real-life); perhaps, that could be why experiential learning has become relevant in academia [15].

Researcher [11] described experiential learning as ”Joyful Learning of Mathematics”. This is because this mode of instruction makes mathematics practical, real, and intriguing, which would stimulate students’ interest to study mathematics in schools. Researcher [22] also described experiential learning as ”the meaning-making process of the individual’s direct experience”. Thus, by utilising experiential learning in this study, the participants constructed their own knowledge, conceptual understanding, reasoning, and problem-solving skills in solving 3-dimensional problems in trigonometry [22]. This reiterates that experiential learning is student centred, rather than teacher centred as seen in the traditional mode of pedagogy. In addition, experiential learning might provide a better and more effective alternative to the traditional approach. This is because experiential learning might provide the needed interactive mode of learning as participants discussed, investigated, experimented, and justified their knowledge constructs as they learn and solve 3-dimensional problems in collaborative small groups by performing relevant activities. Also, in experiential learning, relevant activities and tasks are administered to students eliciting their conceptual understanding, problem-solving competence, and mathematical reasoning [18].
Researcher [17] established that experiential learning sequentially nurtures students’ understanding leading to better performance of theoretical concepts. Therefore, we envisaged that experiential learning would develop participants’ knowledge and understanding of trigonometry. The participants would then apply their knowledge and understanding of trigonometry in incorporation with mathematical creativity to solve mostly unfamiliar 3-dimensional problems. In addition, [28] assert that experiential learning further increased the motivation and positive attitude of participants to learn mathematics and science. This further reiterates the efficacy of experiential learning in mathematics. Also, [2] informed that experiential learning activities help to fix student’s knowledge in their long-term memory, promote positive attitude for learning mathematical concepts, and nurture students’ critical and creative thinking. In addition, [19] applied Kolb’s learning cycle, an experiential learning model in a study. Data collection and analysis was performed using Kolb’s four stages of learning, namely, planning, action, observations, and reflection. These researchers established that experiential learning through Kolb’s learning cycle enhanced the desire of participants and other dispositions to learn mathematics. Thus, in this study, the we sought to test the efficacy of experiential learning in the context of 3-dimensional trigonometric problems.

In implementing experiential learning in this study, we engaged participants with group discussions, brainstorming, exercises, and relevant hands-on activities in relation to 3-dimensional problems in trigonometry. Importantly, students were encouraged to engage in a productive struggle when finding solutions to 3-dimensional problems and learner-centricity was prioritised. Germane to the implementation of experiential learning is students’ prerequisite knowledge to 3-dimensional problems. These are trigonometric ratios, Pythagoras theorem, sine rule, cosine rule, double-angle identities, compound angle identities, problems in 2-dimensions using trigonometry and algebraic simplifications; these constituted the pre-test administered to both the control group and the experimental group. After the intervention, a post-test was administered. This led us to determine the relationship between participants’ prior knowledge and performance in solving 3-dimensional problems in trigonometry. Thus, pretest and posttest scores were modelled using the pretest marks as a proxy for prior knowledge and the test marks (posttest) as a proxy for performance. These were carried out in this study with the aim of testing the efficacy of experiential learning in solving 3-dimensional problems in trigonometry. To achieve this aim, we formulated the following critical research question: (1) How did the two intervention approaches influence the performance of the participants in solving three-dimensional problems?

2. Theoretical framework

In this study, Kolb’s experiential learning model was utilised as the theoretical framework [16]. This theory consists of four distinct stages, namely, concrete
experience, reflective observation, abstract conceptualisation, and active experimentation; these are shown in Figure 1. This model directs how learning should be conducted. The concrete experience stage depicts the foundation of learning. Students are expected to engage with the learning materials personally so that they can obtain first-hand experience as they construct their own knowledge [26]. At this stage, students’ prior knowledge, mathematical creativity, and open-mindedness to be receptive to ideas from peers are integral as they seek for new knowledge [26]. At the reflective observation stage, based on students’ observable experiences they encountered at the first stage, they then thoroughly reflect on those experiences, trying to make sense of those experiences. From this point, students start developing meaning and understanding based on their observable experiences. At this stage, a teacher’s ability to create a conducive learning environment and to select relevant tasks to invoke students’ reasoning as they construct new mathematical knowledge is crucial [15, 26].

Figure 1. Kolb’s experiential learning cycle: adopted from Speer, 2022, p.2

The abstract conceptualisation stage constitutes assimilation of constructed meaning and understanding as they sought to put their observable experiences into practice. Students can intuitively, creatively, and logically relate one idea to the other. They can put their constituted ideas developed in parts into a whole, that is, making generalisations about the main concept. Students, at this stage, are expected to be able to justify their knowledge constructed and solutions to a given problem. The active experimentation stage comprises of testing new ideas, hypothesis, and predictions through practical applications as they put what they learnt into practice leading to new experiences and learning. The conceptualised new knowledge of the students is also tested through active experiments [15, 26].
3. Methodology

3.1. Research design.

A case study research design was considered appropriate for this study. This resulted from the idea that in testing the efficacy of experiential learning, the researchers found that it was imperative to study the participants extensively within an appreciable duration. Furthermore, by providing the researchers with detailed accounts of what transpired in the research field, a case study helped the researchers to obtain substantial data that allowed the researchers to adequately answer the research questions this study sought to investigate [6].

3.2. Participants.

A high school in the Northern Cape Department of Education was purposively sampled to serve as the research field of this study. This is because the researchers needed a school that will allow them easy mobility, having maths learners and relevant instructional tools such as a tablet which is not available in every school. Thus, maths learners in grade 12 consisting of two classes-12A and 12B at the identified school were targeted as participants. These learners had the same mathematics teacher. The students in 12A class consisting of 35 participants were assigned to the control group and were taught using the traditional method. On the other hand, the 12B students were assigned to the experimental group that consisted of 37 participants, and they were taught using the experiential learning approach. The same teacher taught both groups how to solve 3-dimensional trigonometry problems during this study to eliminate the variability that could arise from teaching skills from different teachers. Race, ethnicity, and social background of the learners were not determinant factors in identifying the participants.

3.3. Ethical Consideration.

Relevant ethical procedures were followed in this study, namely, informed consent—permission from appropriate authorities/personnel were obtained; voluntary participation (i.e. only learners who availed themselves willingly were admitted as participants); and confidentiality—participants were anonymous (i.e. their nomenclature were not made known to any third party).

3.4. How experiential learning was implemented in this study.

In this study, Kolb’s experiential learning cycle was applied as follows: (1) At the stage of concrete experience (doing), the instructional activities carried out include participants making critical observations of related examples and problems of solving two-dimensional problems in trigonometry on a tablet. Also, activity sheets on 2-dimensional problems were given to participants. On these activity sheets, participants could dismantle the constituent parts of 2-dimensional problems and to put the constituent parts together. (2) At the reflective observation...
(observing) stage, participants carried out group discussions as they brainstormed on how they can solve non-routine 2-dimensional problems in trigonometry. (3) At the abstract conceptualisation (thinking) stage, participants were sequentially introduced to 3-dimensional problems arranged in order of difficulty. At this stage participants sought to make generalisations on solving 3-dimensional problems in trigonometry. (4) At the active experimentation stage (planning), exercises and homework on solving 3-dimensional problems in trigonometry that require advanced application of knowledge of trigonometry, creative, and reflective thinking were issued to participants.

4. Main results

4.1. Data analysis framework.

A general statistical framework commonly used in literature and in practice to assess the differences between two or more population means or to model the impact of one variable on another is generalised linear models (GLM). Unlike Analysis of Variance (ANOVA) model, the GLM framework is more flexible with the linearity and normality assumptions imposed on the model structure and the data generating process. GLM assumes that the data generating process comes from the exponential family of distribution, thus one has a choice of distributional assumptions to choose from when the normality assumption fails. In addition, the framework does not assume a linear structure between the dependent variable and the independent variables, although it assumes a linear relationship between the transformed response in terms of the link function and the explanatory variables. Therefore, it can detect non-linear differences between two or more population means. In the case of our data, since they were recorded on the count scale, invoking the central limit theorem may not be appropriate.

Alternative distributions that could model the data include Poisson and the negative binomial distributions. As was the case in this study, independent measured controlled experiments are prone to the impact of nuisance factors (random effects). Such nuisance factors may have commonly arisen from factors such as differences in intelligence, barriers to learning, and variations in learning abilities of the participants. It is therefore imperative that we account for these effects, in the modelling framework. The appropriate models that can handle count data while accounting for random effects are the Poisson mixed effect and negative binomial mixed effect models [13].

In deciding on the best model among the two indicated, the overdispersion test was employed. Sequentially generated numbers were assigned to the students who participated in the study. These numbers and the ages of the students were used to account for the random effects in the models. [14] argues that significance levels in the 0.05 and 0.01 neighbourhood provide modest evidence against the null hypothesis, thus the 0.05 and 0.01 significance levels were used. In this study,
our objective is to investigate whether the experiential method of teaching improves students’ performance in solving 3-dimensional problems in trigonometry in comparison to the traditional method of teaching. Furthermore, we investigated the impact of prior knowledge of 3-dimensional problems in trigonometry (from grade 11 or lower grades) on the performance of solving 3-dimensional problems in trigonometry irrespective of the teaching method used. In achieving the first objective, results from the appropriate mixed-effect model were used to test the following hypotheses:

(1) Post and pre-test marks for control (traditional method) groups are different.
(2) Post- and pre-test marks for experimental (experiential method) groups are different.
(3) Post-test marks for the control and experimental groups are different.

In the case of the second objective, using an appropriate mixed-effects model, the relationship between prior knowledge and performance in solving 3-dimensional problems in trigonometry was modelled using the pre-test marks as a proxy for prior knowledge and the post-test marks of the two teaching methods employed in this study as a proxy for performance. Data used in this study were grouped into two. In modelling data with mixed effect models, the power of test is sensitive to the sample size and the dispersion parameter [20]. Through empirical tests, [20] has shown that for a sample size of 50, the power of the Wald, likelihood ratio, and score tests for a negative binomial model are approximately 100% when the dispersion parameter is closer to 0.5. In our case, the combined sample size is 72 and the dispersion parameter is 0.7384 (see Table 2), thus we do not anticipate any problems for our models in detecting significant differences between the marks of the control and the experimental groups. Details of the results, interpretations, and analysis are given in the next section.

4.2. Data Visualisation.

The variability of the pre-test and post-test marks for the control and experimental groups is visually summarised in Figure 2. It appears from the figure that there are some clear differences among the mean marks of the pre-control, pre-treatment, and post-treatment as well as between the mean marks of the post-control, pre-treatment, and post-treatment. Although observation from Figure 3 seems to suggest a linear relationship between prior knowledge and performance, mixed-effects count models are appropriate in modelling the differences between the test marks for the two groups and the relationship between prior knowledge and performance. This is because the residuals from the fitted linear model (as indicated in Table 1) are not normally distributed in addition to the fact that the marks for students recorded in the tests are count data by definition [12]. The choice of the modelling framework is also supported by Figure 4. This is because a plot of the logarithm of post-test marks against
pre-test marks suggests a linear relationship between the two variables which indirectly implies an exponential relationship between the two test marks. In the data analysis framework section, it was argued that since the data is measured on a count scale, it may not be normally distributed. To confirm this, we performed normality tests. In this sense, a normal quantile plot for the aggregated data as well as the Jarque-Bera and Shapiro-Wilk tests were used. The two tests involve the following hypothesis: \( H_0 : \text{The data is normally distributed} \) versus \( H_1 : \text{The data is not normally distributed} \).

| Data           | Test              | Test Statistic | \( Pr(> |z|) \) |
|----------------|------------------|----------------|----------------|
| Original data  | Shapiro           | 0.991          | 0.531          |
|                | Jarque – BeraTest| 0.324          | 0.8505         |
| Residuals      | Shapiro           | 0.973          | 0.122          |
|                | Jarque – BeraTest| 1.6184         | 0.4452         |

When the \( p - value \) of a test is less than 0.01 or 0.05 the null hypothesis is not rejected. Table 1 summarises the test results. In Table 1, the two tests provide converging results. The \( p - values \) of both tests are greater than 0.01 and 0.05, which suggest the rejection of the null hypotheses of normality, thus confirming
4.3. Over-dispersion tests.
When deciding among which generalised mixed-effects models to be used, it is a frequent practice to test for the nature of dispersion in the data. The overdispersion test of [5] was used. The test involves the following hypothesis. The null hypothesis implies that the data is not over-dispersed, while the alternative hypothesis implies that the data are over-dispersed.

**Table 2. Over dispersion test**

| Data       | Dispersion parameter | Test Statistic | Pr(>|z|) |
|------------|----------------------|----------------|----------|
| Original   | 1.7384               | 4.075          | 2.3e-05  |
| Combined   | 0.6393               | -3.567         | 0.9998   |

If the $p$-value $> 0.05$, we do not reject the null hypothesis and conclude that the data is not overdispersed. Table 2 presents the results of the test. It is clear from the table that the data are over-dispersed for the original data (i.e. data for the individual groups) but equidispersed (not over dispersed) for the combined data (data with no distinction between the study groups). This suggests that the negative binomial mixed-effects model is appropriate for testing the differences in the means of the test scores for the groups, while the poison mixed-effects model is appropriate for modelling the relationship between prior knowledge and performance.

4.4. Interpretations of Estimated Models.

**Table 3. Chi-square tests for model diagnostics**

| Model         | Test statistics | Pr(>|z|) |
|---------------|-----------------|----------|
| Negative binomial | 157.0232        | 0.9999   |
| Poisson       | 46.81613        | 0.9813   |

In this section, the estimated parameters, and the associated diagnostic tests of the two estimated models are reported (see Tables 3 to 5) and discussed. The random effect is also significant, suggesting that individual differences significantly affected the results, thus blocking this source of variability was necessary. The Chi-square goodness-of-fit test was used to assess the fitness of the model before making any inferences. The results of the test presented in Table 3 suggest that the two models fit well with the respective data since the $p$-values are higher than the 0.05 level of significance. Therefore, valid inferences can be drawn from the model.
4.4.1. Test of differences in means between the two teaching methods.

Table 4. Pairwise comparison of means

| Pair                        | Estimate | Pr(>|z|) |
|-----------------------------|----------|---------|
| Postcontrol − Precontrol    | 0.2603   | 0.0014  |
| Posttreatment − Pretreatment| 0.2422   | 0.0003  |

As indicated previously, the mixed-effects negative binomial model was used to model the data. In comparing the means of the treatments and the control groups to determine any pairwise significance, the estimated marginal means method by [23] was used. The test is designed for cases where the normality assumption is violated. Thus, it is appropriate for the post hoc analysis in this study since the data used is not normally distributed. The results of a post hoc test for pairwise differences in marginal means are reported in Table 4. It is observed that at 0.05 level of significance, there are significant differences between the pre- and post-test marks for the control group, as well as between the pre- and post-test marks for the experimental group (treatment). This is a clear indication that the two teaching methods, traditional and experiential learning, improved the performance of the participants in solving 3-dimensional problems in trigonometry. However, the experiential method yielded a better improvement in participants’ performance compared to the traditional method. This is because the p-value for differences in mean comparison of the two teaching methods represented by the post-treatment and control variables is less than 0.05. One would query that since the differences in means of the precontrol and pretreatment variables are significant, the improvements in performance by experiential learning might have been caused by differences in learning abilities; however, this source of variability was blocked, hence the improvements in the performance were due to the teaching method.

4.4.2. The relationship between prior knowledge of the participants and performance.

Table 5. Estimated relationship between prior knowledge and performance

| Parameter | Estimate | Std.Error | Pr(>|z|) |
|-----------|----------|-----------|---------|
| Intercept | 2.006942 | 0.081809  | 2e−16   |
| Pretest   | 0.049062 | 0.005920  | 2e−16   |

As mentioned earlier, the relationship between prior knowledge of the participants and their performance was modelled using the mixed-effects Poisson regression
model. Table 5 reports the estimated parameters. The table shows that the standard errors associated with the estimated parameters are quite small, which indicates the precision of the parameters. Furthermore, the estimated parameter for prior knowledge is significant at both 0.05 and 0.01 (because the $p-values$ are less than 0.05 and 0.01), thus, we conclude that prior knowledge of the participants had significant impact on their performances in solving 3-dimensional trigonometry problems regardless of any of the two-teaching methodologies used. Furthermore, ignoring all other factors that may influence performance, when prior knowledge (pre-test marks) of a student improves by 1 point, the expected log count of his or her performance when solving 3-dimensional problems in trigonometry would be expected to increase by approximately 0.05 points (or exponentially by approximately 1.05). This suggest that there is a direct exponential relationship between prior knowledge and performance when solving 3-dimensional problems in trigonometry. This observation is indicated in Figure 5.

5. Conclusion and Recommendation

In summary, the findings of the study were discussed with reference to the research question that the researchers sought to answer, that is, how did the two intervention approaches influence the performance of the participants in solving 3-dimensional problems? Firstly, the outcome of the quantitative data analysis recorded a significant difference between the pre-test marks of the participants and their post-test marks; this was noted in the scores of the two groups under
study. This highlights that the two teaching methods, traditional and experiential learning, improved the performance of participants in solving 3-dimensional problems in trigonometry. However, there was a better improvement in the performance of the participants in the experimental group compared to that of those in the control group. This highlights that experiential learning is an effective mode of pedagogy for teaching students how to solve 3-dimensional problems in trigonometry rather than the traditional approach. Secondly, the analysis conducted in this study established that experiential learning guided students to learn and to perform better than the traditional method when solving 3-dimensional problems in trigonometry. In synopsis, it is within mathematics teachers’ purview to identify and apply effective instructional approaches to teach mathematical concepts. Therefore, the researchers recommend experiential learning to mathematics teachers and students as an effective mode of pedagogy for solve 3-dimensional problems in trigonometry. This is borne out by the fact that the efficacy of experiential learning in solving 3-dimensional problems has been comprehensively established in this study.

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